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CFA LEVEL I

Detailed Formula Sheet



Quantitative Methods

Topic 1 of 10 • Fully Explained

💡 Plain-English explanations 📊 Worked examples 🧠 Memory hooks ⚠️ Exam traps

Time Value of Money

Measuring Returns

Statistical Measures

Probability

Distributions

Sampling & Estimation

Hypothesis Testing

Simple Linear Regression

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★ Mentored by Deepak Goyal — CFA, FRM ★

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👋 **Hey future charterholder!** This sheet doesn't just *list* formulas — it **explains what each one means**, shows a **worked example**, and gives you **memory hooks** so concepts actually stick. Work through it slowly with a pencil. 📖

💡 **In Plain English**

📊 **Worked Example**

🧠 **Memory Hook**

⚠️ **Watch Out**

📌 **Key Fact**

1 💰 Rates & Time Value of Money

🔧 Building Blocks of an Interest Rate

$$r = \text{Real risk-free rate} + \text{Inflation} + \text{Default} + \text{Liquidity} + \text{Maturity premiums}$$

💡 In Plain English

Any required return is a **stack of compensations**. You start with the pure time-value of money (real risk-free rate), then add a premium for every extra risk you take on.

📌 Key Fact

Nominal risk-free rate = Real risk-free rate + Inflation premium (e.g. a T-bill).

🌱 Future Value & Present Value (single sum)

$$FV_N = PV(1 + r)^N \quad | \quad PV = \frac{FV_N}{(1 + r)^N}$$

💡 In Plain English

Money **grows forward** by compounding and **shrinks backward** by discounting. Compounding and discounting are just two sides of the same coin. 🔄

📊 Worked Example

Invest **\$1,000** at **5%** for **3 years**: $FV = 1,000 \times (1.05)^3 = \mathbf{\$1,157.63}$.

Working backward, the PV of \$1,157.63 in 3 yrs at 5% is exactly \$1,000. 🔄

🔄 Effective Annual Rate (EAR)

$$EAR = \left(1 + \frac{r_s}{m}\right)^m - 1 \quad | \quad \text{Continuous: } EAR = e^{r_s} - 1$$

💡 In Plain English

The **more often** interest compounds, the **more you actually earn** in a year. EAR converts any stated rate into one comparable "true" annual number. m = compounding periods per year.

📊 Worked Example

12% compounded **monthly**: $(1 + 0.12/12)^{12} - 1 = \mathbf{12.68\%}$.

12% compounded **continuously**: $e^{0.12} - 1 = \mathbf{12.75\%}$.

🧠 Memory Hook

"Compound **more often** → **earn more often**." More frequency = higher EAR. 📈

17 Annuities & Perpetuity

$$PV_{\text{annuity}} = A \times \frac{1 - (1 + r)^{-N}}{r} \quad | \quad FV_{\text{annuity}} = A \times \frac{(1 + r)^N - 1}{r}$$

$$\text{Annuity Due} = \text{Ordinary Annuity} \times (1 + r) \quad | \quad \text{Perpetuity PV} = \frac{A}{r}$$

In Plain English

An **annuity** is the same cash flow A repeated for N periods. A **perpetuity** never stops (think a bond that pays forever). An **annuity due** pays at the *start* of each period, so it's worth one extra period of growth → multiply by $(1 + r)$.

Worked Example

\$100 per year for 5 years at 6%: $PV = 100 \times [1 - 1.06^{-5}] / 0.06 = \421.24 .

A perpetuity paying \$5 forever at 8%: $PV = 5 / 0.08 = \$62.50$.

Watch Out

Match your **rate and N to the payment frequency**! Monthly payments → monthly rate and number of months, not annual.

2 Measuring Returns

🎯 Holding Period Return (HPR)

$$HPR = \frac{P_1 - P_0 + CF_1}{P_0}$$

In Plain English

Total return over the period = **price gain + income (dividend/coupon)**, divided by what you paid.

Worked Example

Buy at \$50, sell at \$54, collect \$2 dividend: $HPR = (54 - 50 + 2)/50 = 12\%$.

⚖️ Arithmetic vs Geometric vs Harmonic Mean

$$\text{Arithmetic} = \frac{\sum R_i}{n} \quad | \quad \text{Geometric} = [\prod (1 + R_i)]^{1/n} - 1 \quad | \quad \text{Harmonic} = \frac{n}{\sum (1/R_i)}$$

In Plain English

Arithmetic = simple average (best estimate of *next single period*). **Geometric** = compound growth rate (best for *multi-period* performance). **Harmonic** = used for averaging costs, e.g. dollar-cost averaging share prices.

Worked Example

Returns 10%, -5%, 8%: Arithmetic = **4.33%**; Geometric = $[(1.10)(0.95)(1.08)]^{1/3} - 1 = 4.11\%$.

Buying shares at \$10, \$15, \$20 → average cost = $3 / (1/10 + 1/15 + 1/20) = \13.85 .

🧠 Memory Hook

Arithmetic ≥ Geometric ≥ Harmonic (always, when numbers differ & are positive). The bigger the volatility, the bigger the gap between arithmetic and geometric. 🌊

Money-Weighted vs Time-Weighted Return

Money-Weighted = IRR of all cash flows | Time-Weighted = chain of sub-period HPRs

In Plain English

Money-weighted (IRR) is affected by the *timing & size* of your deposits/withdrawals. **Time-weighted** strips out cash-flow timing → it's the **true measure of a manager's skill**, which is why GIPS prefers it.

Memory Hook

"Time-weighted = Talent of the manager." Money-weighted rewards/punishes *your* timing. ⌚

Continuously Compounded & Real Returns

$$r_{cc} = \ln(1 + \text{HPR}) = \ln(P_1/P_0) \quad | \quad (1 + r_{nom}) = (1 + r_{real})(1 + \text{inflation})$$

Worked Example

HPR of 11% → continuous return = $\ln(1.11) = 10.44\%$.

Nominal 8%, inflation 3% → real return = $(1.08/1.03) - 1 = 4.85\%$.

Watch Out

Real return is **not** simply nominal – inflation. That's only a rough approximation — use the exact ratio formula for full marks. ✓

3 Statistical Measures

Variance & Standard Deviation

$$\text{Population } \sigma^2 = \frac{\sum(X_i - \mu)^2}{N} \quad | \quad \text{Sample } s^2 = \frac{\sum(X_i - \bar{X})^2}{n - 1} \quad | \quad \text{SD} = \sqrt{\text{variance}}$$

In Plain English

Variance measures **how spread out** the data is around the mean. Standard deviation is its square root, back in the original units — the headline measure of **total risk**.

Worked Example

Data: 2, 4, 6, 8, 10 (mean = 6). Squared deviations = $16+4+0+4+16 = 40$.

Sample variance = $40/(5-1) = 10$ → sample SD = **3.16**.

Watch Out

A **sample** divides by **$n - 1$** (Bessel's correction); a full **population** divides by N. Picking the wrong one is a classic exam trap. 🐻

Coefficient of Variation & Sharpe Ratio

$$CV = \frac{s}{\bar{X}} \quad | \quad \text{Sharpe} = \frac{\bar{R}_p - R_f}{\sigma_p}$$

In Plain English

CV = risk per unit of return — lets you compare assets of different sizes (**lower CV = better**). **Sharpe** = excess return per unit of total risk (**higher Sharpe = better**).

Worked Example

Portfolio return 12%, risk-free 3%, SD 15%: Sharpe = $(0.12 - 0.03)/0.15 = \mathbf{0.60}$.

Memory Hook

Both are "bang for your buck" ratios: CV is *risk ÷ reward* (want it small), Sharpe is *reward ÷ risk* (want it big). 🌟

Skewness & Kurtosis

Right (positive) skew: Mode < Median < Mean • Left (negative) skew: Mean < Median < Mode

In Plain English

Skew = asymmetry. A long tail to the *right* = positive skew (a few big winners pull the mean up). **Kurtosis** = "fatness of tails." **Excess kurtosis** = kurtosis – 3.

Key Fact

Leptokurtic (excess > 0) = fat tails → more extreme outcomes → more risk. **Platykurtic** (excess < 0) = thin tails. Normal = mesokurtic (excess = 0).

Memory Hook

Skew direction = where the **tail points**, and the **mean chases the tail**. 🐼 "Lepto = Leaping tails (fat & risky)."

Covariance & Correlation

$$\text{Cov}(X, Y) = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{n - 1} \quad | \quad \rho_{XY} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

In Plain English

Covariance tells you the *direction* two variables move together, but its size is hard to read. **Correlation** standardizes it onto a clean scale from **-1 to +1**.

Worked Example

Cov = 0.0096, $\sigma_X = 12\%$, $\sigma_Y = 10\%$: $\rho = 0.0096 / (0.12 \times 0.10) = \mathbf{+0.80}$ (strong positive co-movement).

Key Fact

$\rho = +1$ perfect positive, 0 = no *linear* relation, -1 perfect negative. Diversification benefit grows as **ρ falls below +1**. 🧡

4 Probability Concepts

+ Probability Rules

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\text{Independent: } P(A \text{ and } B) = P(A) \cdot P(B) \quad | \quad \text{Conditional: } P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Worked Example

$P(A) = 0.4$, $P(B) = 0.3$, $P(A \text{ and } B) = 0.1$: $P(A \text{ or } B) = 0.4 + 0.3 - 0.1 = \mathbf{0.60}$.

Watch Out

Don't forget to **subtract the overlap** $P(A \text{ and } B)$ — otherwise you double-count it.

Expected Value & Variance of a Random Variable

$$E(X) = \sum P(x_i) \cdot x_i \quad | \quad \sigma^2(X) = \sum P(x_i) \cdot [x_i - E(X)]^2$$

In Plain English

Expected value = the **probability-weighted average** of all outcomes — your best single guess.

Worked Example

Returns 10%, 20%, -5% with probabilities 0.3, 0.5, 0.2:

$E(R) = 0.3(10) + 0.5(20) + 0.2(-5) = \mathbf{12\%}$.

Portfolio Expected Return & Variance (2 assets)

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2)$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2$$

In Plain English

Portfolio return is a simple weighted average — but **portfolio risk is NOT**. The cross term with ρ is the magic of diversification: low correlation shrinks total risk.

Worked Example

$w_1=0.6$, $\sigma_1=20\%$; $w_2=0.4$, $\sigma_2=10\%$; $\rho=0.3$:

$\sigma_p^2 = 0.36(0.04) + 0.16(0.01) + 2(0.6)(0.4)(0.3)(0.20)(0.10) = 0.01888$

→ $\sigma_p = \sqrt{0.01888} = \mathbf{13.74\%}$ (less than the 16% weighted-average SD — diversification at work!).

Bayes' Formula

$$P(\text{Event} | \text{Info}) = \frac{P(\text{Info} | \text{Event})}{P(\text{Info})} \times P(\text{Event})$$

In Plain English

Bayes **updates** a prior belief after new evidence arrives. New info → revised probability.

Worked Example

A firm defaults 5% of the time. A warning signal fires 90% before defaults, but also 10% falsely. $P(\text{warning}) = 0.9(0.05) + 0.1(0.95) = 0.14$.

$P(\text{default} | \text{warning}) = 0.045 / 0.14 = \mathbf{32.1\%}$.

Counting: Combinations vs Permutations

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!} \quad (\text{order doesn't matter}) \quad | \quad {}_nP_r = \frac{n!}{(n-r)!} \quad (\text{order matters})$$

Worked Example

Choosing 2 from 5: ${}_5C_2 = \mathbf{10}$ ways; ordering 2 of 5: ${}_5P_2 = \mathbf{20}$ ways.

Memory Hook

"**P**ermutation = **P**osition matters." Combination = just the *combo*, order ignored. 🏠

5 Probability Distributions

The Normal Distribution & Z-Score

$$Z = \frac{X - \mu}{\sigma}$$

In Plain English

The **z-score** tells you how many standard deviations a value sits from the mean — letting you read probabilities off the standard normal table.

Key Fact (memorize!)

≈ **68%** of data within $\pm 1\sigma$, ≈ **95%** within $\pm 1.96\sigma$, ≈ **99%** within $\pm 2.58\sigma$. A normal curve is fully defined by just **μ** and **σ** .

Worked Example

Score $X = 80$, $\mu = 70$, $\sigma = 5$: $Z = (80 - 70)/5 = \mathbf{2.0} \rightarrow 80$ is two SDs above average.

Roy's Safety-First Ratio

$$\text{SFRatio} = \frac{E(R_p) - R_L}{\sigma_p} \rightarrow \text{pick the portfolio with the HIGHEST SFRatio}$$

In Plain English

It's basically a Sharpe ratio but using a **minimum acceptable threshold** R_L instead of the risk-free rate. The highest ratio = **lowest probability of falling below your floor**.

Worked Example

Threshold $R_L = 2\%$. Port A: $(12 - 2)/15 = 0.67$; Port B: $(10 - 2)/10 = 0.80$. → **Choose Port B** (higher SFRatio = safer).

Memory Hook

"Safety-First → **Floor first**." Subtract the floor R_L , then maximize. 🏠

Binomial & Lognormal

$$P(X = x) = {}_n C_x \cdot p^x (1 - p)^{n-x} \mid \text{Mean} = np \mid \text{Variance} = np(1 - p)$$

In Plain English

Binomial = count of "successes" in n independent yes/no trials (up/down moves). **Lognormal** is used to model *asset prices* because it can't go negative and is right-skewed.

Worked Example

Probability of exactly 3 "up" days out of 5 when $p = 0.5$:
 ${}_5 C_3 \times 0.5^3 \times 0.5^2 = 10 \times 0.125 \times 0.25 = \mathbf{31.25\%}$.

Key Fact

If continuously compounded returns are **normally** distributed, then prices are **lognormally** distributed. 📈

6 Sampling & Estimation

Standard Error & Confidence Interval

$$\text{Standard Error} = \frac{\sigma}{\sqrt{n}} \quad (\text{or } s/\sqrt{n}) \mid \text{CI: } \bar{X} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

In Plain English

The **standard error** is the SD of the *sample mean* — how much your average would wobble across different samples. Bigger n → smaller error → tighter estimate. 📊

Key Fact (z-values)

90% → 1.645 | **95% → 1.96** | **99% → 2.58**. Use the **t-distribution** when σ is unknown and the sample is small.

Worked Example

$\bar{X} = 50$, $s = 10$, $n = 100$ → $SE = 10/\sqrt{100} = 1$. 95% CI = $50 \pm 1.96(1) = \mathbf{[48.04, 51.96]}$.

Central Limit Theorem (CLT)

For $n \geq 30$, the sampling distribution of $\bar{X}$ is approximately NORMAL — whatever the population shape

In Plain English

This is the engine behind nearly all of inferential statistics: take big-enough samples and the mean behaves normally, so we can build z-based confidence intervals and tests. ⚙️

Memory Hook

"30 is the magic number" — the usual cutoff for a "large" sample. ✨

7 Hypothesis Testing

Test Statistic (single mean)

$$t = \frac{\bar{X} - \mu_0}{s / \sqrt{n}} \quad (df = n - 1)$$

In Plain English

You measure how far your sample result is from the "null" claim, in standard-error units. If it's far enough (beyond the critical value), you **reject** H_0 .

Worked Example

$\bar{X} = 52$, $\mu_0 = 50$, $s = 10$, $n = 25 \rightarrow SE = 10/5 = 2$; $t = (52 - 50)/2 = \mathbf{1.0}$. Critical t (df 24, two-tail 5%) $\approx 2.064 \rightarrow$ **fail to reject** H_0 .

Watch Out

Reject H_0 only when **|test stat| > critical value** (or $p\text{-value} < \alpha$). Match one-tail vs two-tail to the alternative hypothesis!

Type I vs Type II Errors

Type I = reject a TRUE H_0 (prob = α) • Type II = fail to reject a FALSE H_0 (prob = β)

Key Fact

Power of a test = $1 - \beta$ = probability of correctly rejecting a false H_0 . Lowering α (fewer false alarms) tends to raise β (more misses).

Memory Hook

Type I = a false **alarm** 🚨 (you cried wolf). Type II = a **miss** 😞 (real wolf, no alarm).

8 Simple Linear Regression

The Regression Line

$$\hat{Y} = b_0 + b_1X \quad | \quad b_1 = \frac{\text{Cov}(X,Y)}{\sigma^2_X} \quad | \quad b_0 = \bar{Y} - b_1\bar{X}$$

In Plain English

Fits the "best straight line" through the data. The **slope** b_1 = how much Y changes per 1-unit change in X (the independent variable). The line always passes through $(\bar{X}, \bar{Y})$.

Worked Example

$\text{Cov}(X,Y) = 12$, $\text{Var}(X) = 8 \rightarrow b_1 = 1.5$. With $\bar{X} = 5$, $\bar{Y} = 10 \rightarrow b_0 = 10 - 1.5(5) = 2.5$.

Equation: $\hat{Y} = 2.5 + 1.5X$.

Goodness of Fit: R^2 & SEE

$$R^2 = \frac{\text{Explained variation (RSS)}}{\text{Total variation (SST)}} = 1 - \frac{\text{SSE}}{\text{SST}} \quad | \quad \text{SEE} = \sqrt{\frac{\text{SSE}}{n-2}}$$

In Plain English

R^2 = the % of Y's variation explained by X (0 to 1; higher = better fit). **SEE** = typical size of the prediction errors (lower = tighter fit). In *simple* regression, $R^2 = (\text{correlation})^2$.

Worked Example

If correlation $\rho = 0.80$, then $R^2 = 0.80^2 = 0.64 \rightarrow X$ explains 64% of Y's variation.

Key Fact

$\text{SST} = \text{RSS} + \text{SSE}$ (Total = Explained + Unexplained). Total variation always splits into these two buckets. 