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CFA LEVEL I

Detailed Formula Sheet



Derivatives

Topic 7 of 10 • Fully Explained

💡 Plain-English explanations 📊 Worked examples 🧠 Memory hooks ⚠️ Exam traps

Derivatives Basics

Arbitrage & Cost of Carry

Forwards & Futures

Swaps & FRAs

Option Payoffs

Put-Call Parity

Binomial Pricing

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👋 **Topic 7 — Derivatives!** Contracts whose value **derives** from an underlying asset. The whole topic rests on one idea: **no-arbitrage pricing** — if two things have the same future payoff, they must cost the same today. Each card has the formula, a plain-English meaning, a worked example, and **memory hooks**. 🗂️

💡 **In Plain English** 📖 **Worked Example** 🧠 **Memory Hook** ⚠️ **Watch Out** 🔑 **Key Fact**

1 🗂️ Derivatives Basics

🌱 Forward Commitments vs Contingent Claims

Forward Commitments: forwards • futures • swaps (obligation) | Contingent Claims: options (a right)

💡 In Plain English

A **forward commitment** locks both sides into a trade — symmetric payoffs, zero cost to enter. A **contingent claim** (option) gives the buyer a *right* but not an obligation — asymmetric payoff, paid for with an upfront premium.

🔑 Exchange-Traded vs OTC

Exchange-traded (futures, many options): standardized, cleared, low credit risk. **OTC** (forwards, swaps): customized, flexible, but carry counterparty credit risk.

2 ⚖️ Arbitrage, Replication & Cost of Carry

🏗️ Pricing a Forward by Cost of Carry

$$F_0(T) = S_0 \times (1 + r)^T$$

With carry: $F_0 = [S_0 + \text{PV}(\text{costs}) - \text{PV}(\text{benefits})] \times (1 + r)^T$

💡 In Plain English

The forward price = today's spot grown at the risk-free rate (the cost of "carrying" the asset to the future). **Storage costs raise** the forward price; **income/benefits** (dividends, coupons, convenience yield) **lower** it.

📖 Worked Example

S_0 \$100, r 5%, T 1 yr $\rightarrow F_0 = 100 \times 1.05 = \text{\$105}$. If the asset pays \$3 income $\rightarrow F_0 = 105 - 3 = \text{\$102}$.

🧠 Memory Hook

"Carry **COSTS** push the forward **UP**; carry **BENEFITS** pull it **DOWN**." Spot, grown at r , adjusted for carry. 🔑

3 Forwards & Futures

Payoffs & Value Over Time

Long forward payoff at $T = S_T - F_0(T)$ | Short = $F_0(T) - S_T$

$$\text{Value to long during life: } V_t = \frac{F_t(T) - F_0(T)}{(1 + r)^{(T-t)}}$$

In Plain English

A forward costs nothing to enter ($V_0 = 0$). As prices move, it gains or loses value: the long profits when the new forward price exceeds the locked-in price. At expiry, the payoff is simply spot minus the agreed forward price.

Worked Example

Locked-in $F_0 = \$105$; at expiry $S_T = \$112 \rightarrow$ long payoff = $112 - 105 = \text{+\$7}$ (short loses \$7).

Futures vs Forwards

Futures: exchange-traded, standardized, daily mark-to-market | Forwards: OTC, customized, settle at maturity

In Plain English

Futures are **marked-to-market daily** — gains/losses settle into a margin account each day, and a clearinghouse removes most credit risk. Forwards settle only at maturity, so credit exposure builds up over the life of the contract.

Key Fact

If interest rates are **positively correlated** with the futures price, futures prices slightly exceed forward prices (daily gains get reinvested at higher rates); if uncorrelated, the two are effectively equal.

4 Swaps & FRAs

A Swap = a Series of Forwards

Plain-vanilla interest rate swap: exchange **FIXED** for **FLOATING** payments on a notional

In Plain English

A swap is just a bundle of forward contracts settling on many dates. The **swap (fixed) rate** is set so the contract is worth **zero at initiation** — the PV of fixed payments equals the PV of expected floating payments.

Key Fact

A **Forward Rate Agreement (FRA)** is a single forward on an interest rate: one party locks in a borrowing/lending rate for a future period. A swap is essentially a chain of FRAs.

Memory Hook

"One forward = FRA; many forwards = swap." Same no-arbitrage DNA, just more settlement dates.

5 Options: Payoffs, Moneyness & Value

Payoffs & Profit

$$\text{Call payoff} = \text{Max}(0, S_T - X) \mid \text{Put payoff} = \text{Max}(0, X - S_T)$$

$$\text{Profit} = \text{Payoff} - \text{Premium paid (call: } -c_0; \text{ put: } -p_0)$$

In Plain English

A **call** buyer wins when the price rises above the strike; a **put** buyer wins when it falls below. The buyer's loss is capped at the premium; the seller's gain is capped at the premium (but their loss can be large).

Worked Example

Call, X \$50, S_T \$58, premium \$3 → payoff = $\text{max}(0, 8) = \$8$; profit = **\$5**. Put, X \$50, S_T \$44 → payoff = **\$6**.

Moneyness & Option Value

$$\text{Call ITM if } S > X \bullet \text{Put ITM if } S < X \bullet \text{ATM if } S = X$$

$$\text{Option value} = \text{Intrinsic value} + \text{Time value}$$

In Plain English

Intrinsic value = what the option is worth if exercised now (never below zero). **Time value** = the extra premium for the chance it moves further into the money before expiry; it decays to zero at expiration.

Value Drivers

Call value rises with: ↑ underlying, ↓ strike, ↑ volatility, ↑ time, ↑ risk-free rate. **Higher volatility raises BOTH call and put values** (more upside, downside still capped).

6 Put-Call Parity & Option Bounds

Put-Call Parity (& Forward Version)

$$S_0 + p_0 = c_0 + X(1 + r)^{-T} \quad (\text{protective put} = \text{fiduciary call})$$

$$F_0(T)(1 + r)^{-T} + p_0 = c_0 + X(1 + r)^{-T}$$

In Plain English

A **protective put** (own the asset + a put) and a **fiduciary call** (own a call + a risk-free bond paying X) give the identical payoff — so by no-arbitrage they must cost the same. Rearrange to find any missing piece.

Worked Example

S_0 \$50, p_0 \$5.36, X \$55, r 5%, T 1: $c_0 = 50 + 5.36 - 55/1.05 = 50 + 5.36 - 52.38 = \text{\$2.98}$.

Memory Hook

"Stock + Put = Call + Bond." Both protect the downside while keeping the upside — same payoff, same price.


No-Arbitrage Option Bounds

$$\text{Call: } \text{Max}[0, S_0 - X(1+r)^{-T}] \leq c_0 \leq S_0$$

$$\text{Put: } \text{Max}[0, X(1+r)^{-T} - S_0] \leq p_0 \leq X(1+r)^{-T}$$

 In Plain English

A call can never be worth more than the stock itself, nor less than its discounted intrinsic value. A put can never exceed the present value of the strike. These bounds keep prices arbitrage-free.

7

Binomial Option Pricing

The One-Period Binomial Model

$$\text{Risk-neutral prob: } \pi = \frac{(1+r) - d}{u - d} \quad (u, d = \text{up/down gross returns})$$

$$c_0 = \frac{\pi \cdot c_1^+ + (1 - \pi) \cdot c_1^-}{(1 + r)^T}$$

 In Plain English

Assume the underlying can only go up ($\times u$) or down ($\times d$). Compute the **risk-neutral probability** π , take the probability-weighted option payoff, and discount at the risk-free rate. **Real-world probabilities don't matter!**

 Worked Example

S_0 \$50 $\rightarrow$ \$60 ($u=1.2$) or \$40 ($d=0.8$), X \$55, r 5%:

$$\pi = (1.05 - 0.8)/(1.2 - 0.8) = 0.25/0.40 = \mathbf{0.625}.$$

$$c_0 = [0.625 \times \$5 + 0.375 \times \$0]/1.05 = 3.125/1.05 = \mathbf{\$2.98}.$$

 Watch Out

π is a **risk-neutral** (synthetic) probability, not the real chance of an up move. The actual probability q never enters the option price — only u , d , and r do.