

CFA® Level I

One-Page Learning Module Summaries

Fixed Income

19 learning modules · one page each · built from CFA Program Level I curriculum, Volume 6

How each page works

■ Module banner	Exact module title, exam relevance (High/Medium/Low-yield) and whether it is calculation- or concept-heavy.
■ The big picture	Two or three sentences of intuition: why the module exists.
■ Learning outcomes	Every LOS of the module, in book order, rewritten as what you must know or do.
■ Key formulas / key facts	Formulas in curriculum notation with every variable defined; fact tables for concept modules.
■ Worked example	One numerical example (or a short Ethics case). Tagged [CFA curriculum] when taken from the curriculum, [Illustrative] when made up by RBei.
■ What examiners like to test	Traps, key distinctions and typical question angles.

Notes. (1) The curriculum lists LOS without letters; “LOS 1, 2, 3” follows the order in the book. (2) Exam relevance is RBei’s judgement from the number of LOS, the calculation LOS and the weight of worked examples and practice problems in each module; the curriculum does not rate modules. (3) [India context, not curriculum] marks Indian examples added for context; they are not curriculum. (4) [Note: ...] marks places where the curriculum is inconsistent or offers two framings.

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Module 1 — Fixed-Income Instrument Features

Topic: Fixed Income · Exam relevance: **Medium-yield** · **Concept-heavy**

The big picture

A bond is a contract: the issuer borrows the principal today and promises coupons and repayment on set dates. Every later module — pricing, yield, duration, credit — starts from the features written into that contract (issuer, maturity, par, coupon, seniority, options) and from the indenture that says what the issuer must and must not do while the money is outstanding.

Learning outcomes in plain language

- LOS 1 Describe the features of a fixed-income security:** issuer (sovereign, local, supranational, quasi-government, corporate, special purpose entity), maturity/tenor (money market ≤ 1 year vs capital market > 1 year; perpetuals), principal (par/face value), coupon rate and frequency (fixed, floating = MRR + credit spread, zero-coupon), seniority (senior vs junior/subordinated) and contingency provisions (call, put, conversion); plus current yield, YTM and the yield curve.
- LOS 2 Describe the contents of a bond indenture** (features, sources of repayment, collateral and credit enhancements, covenants) **and contrast affirmative covenants** (what the issuer *must* do) **with negative covenants** (what the issuer *must not* do).

Key formulas

$$\text{Coupon per period} = \frac{\text{Coupon rate} \times \text{Par value}}{\text{Payments per year}} \quad \text{FRN coupon rate} = \text{MRR} + \text{Credit spread} \quad \text{CY}_t = \frac{\text{Annual coupon}_t}{\text{Bond price}_t}$$

$$\text{YTM (IRR): } PV = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \dots + \frac{PMT + FV}{(1+r)^N}, \quad \text{annual YTM} = r \times \text{periods per year}$$

PMT = coupon per period; FV = par; N = number of periods; r = periodic yield. YTM equals the realised return only if the investor (1) receives all payments (no default), (2) holds to maturity and (3) reinvests every coupon at the YTM. MRR resets each period; the credit spread (in bps) is fixed at issuance. Spread vs government = issuer YTM – sovereign YTM (compensation for credit risk).

Key facts: indenture and covenants

Repayment	Sovereign: taxing (and money-printing) power. Local/regional: taxes or project fees (tolls, transit). Corporate: operating cash flows; secured bonds add a claim (lien/pledge) on collateral as a <i>secondary</i> source. ABS: cash flows of a pool of loans/receivables, split into tranches.
Affirmative	Use of proceeds, timely financial reports, redemption at a premium if acquired, <i>pari passu</i> (equal footing with debt of the same seniority), cross-default. Administrative; low cost to the issuer.
Negative	Limits on liens, sale-and-leaseback, merger/asset sales, additional debt, dividends/buybacks (often unless an <i>incurrence test</i> is met), investments, asset disposals; <i>negative pledge</i> (no new debt senior to existing). Debt restriction test (VIVU), e.g. net debt/EBITDA $\leq 5.0\times$, coverage $> 2.5\times$.

Worked example (CFA curriculum)

BRWA 3.2% five-year notes, USD300 million, semiannual. Annual interest = $300,000,000 \times 3.2\% = \text{USD}9,600,000$, so each coupon is USD4,800,000 and the final payment is USD304.8 million.

Current yield at a price of 101: $\text{CY} = 3.2/101 = 3.168\%$. **YTM** at 101: solve $101 = \sum_{t=1}^{10} 1.6/(1+r)^t + 100/(1+r)^{10} \Rightarrow r = 1.49\%$ per half-year, i.e. $1.49\% \times 2 = 2.98\%$ annualised (below the 3.2% coupon because the bond trades at a premium). On the curriculum's yield-curve exhibit (bond at par, YTM 3.2%) versus the 2.3% five-year Treasury, the credit spread is $3.2\% - 2.3\% = 90 \text{ bps}$.

FRN (Antelas AG, EUR250 million, MRR + 250 bps, quarterly): with MRR = 0.25% in the last quarter, coupon = 2.75% and the maturity payment = $250,000,000 \times 2.75\%/4 + 250,000,000 = \text{EUR}251,718,750$.

What examiners like to test

- **Coupon on par, not price:** coupon = coupon rate $\times$ par; the *yield* depends on price. If the price falls, CY and YTM rise.
- **FRN spread is fixed:** if the issuer's credit worsens, the FRN coupon does *not* change — only MRR resets (the price falls instead). A negative MRR simply lowers the coupon.
- **Secured $\neq$ ABS:** a secured corporate bond still relies primarily on operating cash flows; collateral is a secondary source. ABS rely on the pooled assets.
- **Covenant classification:** cross-default and *pari passu* are *affirmative*; debt limits, coverage minimums, lien limits and the negative pledge are *negative*. Overly strict covenants can force an avoidable default.

Module 2 — Fixed-Income Cash Flows and Types

Topic: Fixed Income · Exam relevance: **Medium-yield · Calculation + concept**

The big picture

Bonds differ mainly in *when* principal comes back (bullet, amortizing, sinking fund, waterfall), *how* interest is set (fixed, floating, step-up, index-linked, zero, deferred, PIK) and *who holds the options* (issuer: call; investor: put, conversion). Where and in which currency a bond is issued decides its legal, regulatory and tax treatment.

Learning outcomes in plain language

LOS 1 Describe common cash flow structures — bullet, fully and partially amortizing (balloon), sinking fund, waterfall; floating-rate, step-up, credit-linked, PIK, inflation-linked (capital-indexed vs interest-indexed), zero-coupon and deferred coupon — **and contrast contingency provisions** that benefit the *issuer* (fixed-price call, make-whole call) with those that benefit the *investor* (put, conversion), plus warrants and CoCos.

LOS 2 Describe how legal, regulatory and tax considerations affect issuance and trading: domestic vs foreign bonds vs Eurobonds vs global bonds (and sukuk); taxation of coupon income, capital gains and original issue discount (OID).

Key formulas

$$A = \frac{r \times \text{Principal}}{1 - (1 + r)^{-N}} \quad \text{Conversion ratio} = \frac{\text{Par value}}{\text{Conversion price}} \quad \text{Conversion value} = \text{Conversion ratio} \times \text{Share price}$$

$$\text{Inflation-adjusted principal} = \text{Principal} \times \left(1 + \frac{CPI_{\text{new}} - CPI_{\text{old}}}{CPI_{\text{old}}} \right) \quad \text{OID} = \text{Par value} - \text{Issue price}$$

A = level periodic payment; r = market rate *per period* (annual rate/periods per year); N = number of periods. Interest each period = $r \times$ opening balance; principal repaid = $A -$ interest. Partially amortizing (balloon B): amortize $\text{Principal} - B/(1 + r)^N$ and pay B at maturity.

Key facts: who benefits?

Callable (issuer)	Refinance if rates fall; call protection period then call schedule. Investor has call risk: price capped near call price, reinvestment risk $\Rightarrow$ <i>higher</i> yield. Behaves option-free when $YTM >$ coupon.
Puttable (investor)	Sell back (usually at par) if rates rise; put price is a floor $\Rightarrow$ <i>lower</i> yield. Behaves option-free when $YTM <$ coupon.
Convertible (investor)	Swap debt for shares; lowest coupon. Tracks straight bond when share price $\ll$ conversion price, tracks conversion value when $\gg$. CoCos convert automatically on the <i>downside</i> (capital-ratio trigger).
Jurisdiction	Domestic (issuer's country); foreign (another country's market and currency); Eurobond (outside any single jurisdiction; USD Eurobonds not sold to US investors at issue, except large qualified institutional investors); global (Eurobond + at least one domestic market). Currency of denomination drives price more than place of issue.

Worked example (CFA curriculum)

(a) Amortizing. BRWA: USD300m, 3.2% p.a., semiannual, 5 years: $r = 1.6\%$, $N = 10$. $A = 0.016 \times 300 / (1 - 1.016^{-10}) = \text{USD}32.70\text{m}$. Period 1: interest $300 \times 1.6\% = 4.80$, principal $32.70 - 4.80 = 27.90$, balance 272.10; period 2 interest = $272.10 \times 1.6\% = 4.35$. With a USD150m balloon: PV of balloon = $150 / 1.016^{10} = 127.98$, amortize $300 - 127.98 = 172 \Rightarrow A = 18.75$, final payment $18.75 + 150 = 168.75$.

(b) TIPS. Par USD115m, 1.00% coupon, CPI 265.00 $\rightarrow$ 267.25: adjustment = 0.849%; principal = $115\text{m} \times 1.00849 = \text{USD}115,976,350$; semiannual coupon = $1.00\% \times 115,976,350 / 2 = \text{USD}579,882$.

(c) Convertible. ZTGB: EUR1,000 par, conversion price EUR42 (50% premium to EUR28): ratio = $1,000 / 42 = 23.81$ shares; at a share price of EUR35, conversion value = $23.81 \times 35 = \text{EUR}833.35 < 1,000$, so hold the bond.

What examiners like to test

- **Amortizing vs bullet:** amortizing and sinking-fund bonds have *lower* credit risk but *higher* reinvestment risk.
- **Yield ordering:** callable $>$ option-free $>$ puttable $>$ convertible (investor pays for options it owns via lower yield).
- **Deflation:** a capital-indexed bond's principal and coupon fall, but investors usually get the greater of adjusted principal or par at maturity.
- **OID tax:** with an OID provision (US) the investor is taxed on a prorated share each year and pays no capital gains tax at maturity; without one, no income until maturity, then capital gains tax.

Module 3 — Fixed-Income Issuance and Trading

Topic: Fixed Income · Exam relevance: Low-yield · Concept-heavy

The big picture

Bond markets are sliced three ways — issuer type (sector), credit quality and time to maturity — and investors pick their spot on that grid to match liabilities or earn more return. Because one issuer has many bonds, bonds mature and new ones keep arriving, bond indexes and bond trading look very different from their equity counterparts: bigger, busier indexes and mostly over-the-counter, dealer-quoted trading.

Learning outcomes in plain language

- LOS 1 Describe fixed-income market segments and their participants:** segments by sector, credit quality (investment grade BBB–/Baa3 and above vs high yield BB+/Ba1 and below; “fallen angels”) and maturity (short <1y, intermediate 1–10y, long >10y); who issues what (sovereigns, IG and HY corporates, securitized issuers) and who buys what (money market funds, pension funds, insurers, bond funds/ETFs, hedge and distressed-debt funds, central banks).
- LOS 2 Describe types of fixed-income indexes:** broad aggregate indexes vs narrower indexes (by sector, credit, maturity, geography, currency, ESG), and how they differ from equity indexes.
- LOS 3 Compare primary and secondary bond markets with equity markets:** debut issuers, public offering vs private placement, underwritten vs best-efforts offerings, shelf registration, reopenings, sovereign auctions; OTC dealer markets, bid–offer spreads, on-the-run vs seasoned bonds, distressed debt.

Key facts

Issuer map	Short term: T-bills, repo, commercial paper (CP), ABCP. IG corporates: unsecured bonds across maturities, CP backed by a 364-day facility plus a revolving credit facility. HY corporates: secured, shorter maturities, leveraged/term loans, no access to unsecured CP.
Investor map	Money market funds (short, IG); pension funds and insurers (long, fixed coupons to match long liabilities); hedge and distressed-debt funds (HY, equity-like returns); central banks (sovereigns, reserves, monetary policy).
Vs equity index	(1) Many more constituents (some >10,000) because one issuer has many bonds; (2) far more turnover (finite maturity, frequent new issues) — usually rebalanced <i>monthly</i> ; (3) weighted by <i>market value of debt outstanding</i> , so heavy in government debt; trackers hold a representative <i>sample</i> .
Index examples	Bloomberg Barclays Global Aggregate (IG only, 28 currencies, fixed-rate, minimum issue size, ≥1y); J.P. Morgan EMBI+ (USD sovereign EM debt rated BBB+/Baa1 or below, ≥USD500m); Euro Corporate Sustainable SRI (IG, EUR, MSCI ESG rating ≥ BBB, business-line exclusions).
Primary market	IG unsecured deals priced within hours (announce, book, guidance, launch, price vs government benchmark). HY/secured: longer, complex covenants, may be best-efforts. Small or customised: private placement. Repeat issuers usually issue a <i>new</i> bond near par; a reopening adds to an existing bond priced away from par.
Secondary market	Mostly quote-driven OTC (platforms: MarketAxess, Tradeweb). Liquidity: on-the-run DM sovereigns (fraction of a bp) > recent IG corporates (a few bps) > seasoned/infrequent issuers (10–20 bps or more).

Worked example (CFA curriculum)

Choosing a benchmark (Baywhite Financial). The Core Bond Fund (3,601 positions) holds AAA 55%, AA 4%, A 13%, BBB 19%, not rated 9%: rated investment grade = $55 + 4 + 13 + 19 = 91\%$, with no high yield. It is diversified by sector (sovereign 28%, corporate 33%, MBS 23%, other ABS 16%) and is mostly intermediate and long term, so a global aggregate IG index fits (a Japanese aggregate index if it invested only in Japan). The Short-Term Bond Fund has $41\% + 57\% = 98\%$ of holdings maturing within three years (all IG or unrated), so it should be judged against a short-term IG index. Neither fund can buy VIVU’s 6.5% callable notes (high yield).

Distressed debt (Hertz, 2020). After the Chapter 11 filing its 2022 and 2028 bonds traded below 10% of par (>90% fall in three months), yet kept trading and recovered to pre-pandemic prices a year later — the equity of a distressed issuer is usually delisted first.

What examiners like to test

- **The BBB–/BB+ line:** BBB– is the lowest investment grade; BB+ is the highest high yield. [Note: One practice-problem solution in this module describes high yield as “BBB– or below”; the module text (and the rest of the curriculum) puts BBB– in investment grade.]
- **Index traps:** bond indexes have *more* constituents and *more* turnover than equity indexes and are harder to replicate fully.
- **Widest bid–offer spread:** a seasoned bond of an infrequent issuer, not a new sovereign or new IG corporate.
- **Debut issuer:** usually replaces private debt (bank loans) with bonds; can be a public offering or a private placement.

Module 4 — Fixed-Income Markets for Corporate Issuers

Topic: Fixed Income · Exam relevance: **Medium-yield · Calculation + concept**

The big picture

Companies and banks fund short-term needs through a ladder of choices that trades cost against reliability: bank lines, secured loans and factoring, commercial paper, deposits, interbank loans and — for financial institutions above all — repos, which are collateralized loans dressed up as a sale and repurchase. For long-term money, credit quality decides almost everything: investment-grade issuers borrow cheaply and flexibly; high-yield issuers accept collateral, covenants and shorter maturities.

Learning outcomes in plain language

- LOS 1 Compare short-term funding alternatives:** for corporations — uncommitted lines (cheapest, least reliable), committed lines (commitment fee, often 364 days), revolvers (multiyear, covenants, most reliable), secured/asset-based loans, assignment vs factoring of receivables, commercial paper (rollover risk, backup lines); for financial institutions — deposits, CDs, interbank and central bank funds, CP and ABCP.
- LOS 2 Describe repurchase agreements:** mechanics, initial margin, haircut and variation margin, uses (financing a security, secured cash investment, borrowing a security to short), reverse repo, general vs special collateral, factors that drive the repo rate, and risks (default, collateral, margining, legal, netting/settlement; bilateral vs triparty).
- LOS 3 Contrast long-term funding of IG vs HY issuers:** YTM mostly benchmark vs mostly spread, bond-like vs equity-like cash flows, few vs many covenants, up to 30y vs typically $\leq 10y$, staggered maturities vs callable bonds/leveraged loans; fallen angels.

Key formulas

$$\text{Repurchase price} = \text{Purchase price}_0 \times \left[1 + \text{Repo rate} \times \frac{\text{days}}{360} \right] \quad \text{Initial margin} = \frac{\text{Security price}_0}{\text{Purchase price}_0}$$

$$\text{Haircut} = \frac{\text{Security price}_0 - \text{Purchase price}_0}{\text{Security price}_0} \quad \text{Variation margin} = (\text{Initial margin} \times \text{Purchase price}_t) - \text{Security price}_t$$

Convention: repo interest is **simple interest**, repo days over a **360-day year** (as in all curriculum examples). Purchase price = cash lent. Positive variation margin: the cash borrower (security seller) must post collateral; negative: over-collateralized, collateral can be released. **Repo rate higher** with: higher money market rates, riskier collateral (equities, EM bonds), longer term, less collateral; **lower** for high-demand (special, e.g. on-the-run) collateral — even negative.

Worked example (CFA curriculum)

Five-year Treasury note, price = face = USD100,000,000; 30-day repo at 0.25%.

(a) **No margin:** repurchase price = $100,000,000 \times [1 + 0.25\% \times 30/360] = \text{USD100,020,833}$ (interest USD20,833).

(b) **102% initial margin:** purchase price = $100,000,000/1.02 = \text{USD98,039,216}$ (the seller finances the other USD1,960,784 elsewhere); haircut = $1,960,784/100,000,000 = 1.96\%$; repurchase price = $98,039,216 \times [1 + 0.25\% \times 30/360] = \text{USD98,059,641}$.

(c) **Variation margin on day 5,** note price rises to 103%; purchase price₅ = $98,039,216 \times [1 + 0.25\% \times 5/360] = 98,042,620$; VM = $1.02 \times 98,042,620 - 103,000,000 = -\text{USD2,996,528} \Rightarrow$ over-collateralized; the borrower can recall notes with face value $\approx 2,996,528/1.03 = \text{USD2.91 million}$.

(d) **Reverse repo to short:** a hedge fund lends USD100m in the repo, sells the note for USD100m, buys it back in 30 days at 99% (USD99m) and receives USD100,020,833: profit = **USD1,020,833** (before costs).

What examiners like to test

- **Who is who:** security seller = cash *borrower* (keeps the coupons and ownership); security buyer = cash *lender*, who calls it a *reverse repo*.
- **Margin vs haircut:** a 103% initial margin gives a haircut of $1 - 1/1.03 = 2.91\%$, not 3%.
- **Reliability ladder:** uncommitted line (no fee, bank may refuse) < committed line (fee) < revolver (multiyear, covenants). CP investors demand backup lines against rollover risk; ABCP is off the sponsoring bank's balance sheet.
- **Fallen angels:** HY credit risk but IG-style bonds (non-callable, few covenants, long maturity); forced selling by IG-only holders hits prices.

Module 5 — Fixed-Income Markets for Government Issuers

Topic: Fixed Income · Exam relevance: **Low-yield** · **Concept-heavy**

The big picture

Governments borrow differently from companies: a sovereign can **tax** (and often print its own currency), so its domestic-currency debt is the lowest-risk debt in its market and becomes the **benchmark** for pricing everything else. Instead of opportunistic, underwritten deals, sovereigns sell debt through **regular public auctions**. Below the sovereign sit agencies, local governments and supranationals, whose credit depends on how predictable their repayment sources are.

Learning outcomes in plain language

- LOS 1 Describe funding choices of sovereigns, non-sovereigns, agencies and supranationals.** *Sovereigns:* DM (stable, diversified, reserve currency, “default-risk-free”) vs EM (less diversified, restricted currencies; domestic vs **external** debt owed to foreign creditors). Fiscal policy sets the **level** of debt; debt management sets its **composition**: T-bills (1–12 months, zero-coupon), notes and bonds (fixed, floating, inflation-linked, foreign currency), plus government-guaranteed MBS. Ricardian equivalence says maturity is irrelevant, but in practice rollover risk makes governments spread debt across maturities. *Non-sovereigns:* agencies (repaid from operations, sovereign backing as secondary source, no full liquidity premium), local governments (**GO bonds** repaid from taxes vs **revenue bonds** repaid from project fees, longer-dated), supranationals (member-state support, highest credit quality).
- LOS 2 Contrast issuance and trading of government vs corporate debt.** Sovereigns use scheduled **auctions** run by the treasury/finance ministry (competitive vs non-competitive bids; single-price vs multiple-price) through designated **primary dealers**; corporates use investment-bank underwriters and issue opportunistically. After issue both trade mainly OTC; on-the-run sovereigns are the most liquid benchmarks; sovereign holders often have “non-economic” motives (central banks, reserve managers, regulated banks and insurers), which lowers sovereign borrowing costs.

Key facts

Long-term issues	(1) risk-free benchmark for every maturity; (2) hedging interest rate risk; (3) preferred collateral in repo and derivatives; (4) monetary policy and FX reserves. Short-term bills carry a liquidity premium (lower yields) and let the treasury flex auction sizes.
Competitive	States price (yield) and quantity; gets nothing if its bid price is below the auction price (bid yield above the cut-off).
Non-competitive	Accepts the auction price and always receives securities.
Single-price	Also called uniform-price. Bids ranked from lowest yield (highest price); the yield that fills the issue is the cut-off (stop) yield ; <i>all</i> winners pay that one price. May lower funding cost and broaden distribution.
Multiple-price	Each winner pays its own bid price; may concentrate allotment in fewer large bids.
Agencies	Borrows near the sovereign yield but does <i>not</i> get the full sovereign liquidity premium.

Worked example (CFA curriculum)

30-year Singapore government bond auction (uniform-price). Offered and allotted: SGD2,600,000,000; non-competitive allotment SGD207,355,000, so competitive bidders received $2,600,000,000 - 207,355,000 = \text{SGD}2,392,645,000$. Total bids SGD4,105,487,000: bid-to-cover = $4,105,487,000 / 2,600,000,000 = 1.58\times$ (“over 1.5×”).

Cut-off yield **1.95%**; rule: coupon = cut-off yield rounded *down* to the nearest 0.125% $\Rightarrow$ coupon **1.875%**. Because the coupon is below the yield, every winner pays the same *discount* price: $\sum_{t=1}^{60} \frac{0.9375}{(1.00975)^t} + \frac{100}{(1.00975)^{60}} = 98.303$ (semiannual). Bidders who bid yields above 1.95% receive nothing; those who bid below still pay 98.303.

India (curriculum box): in July 2021 the RBI moved most government bond auctions (2- to 14-year and floating-rate bonds) from multiple-price to single-price after several partially failed auctions; 30- and 40-year bonds stayed multiple-price.

What examiners like to test

- **Level vs composition:** fiscal policy $\rightarrow$ *level* of debt; debt management $\rightarrow$ *composition* (maturity, type). Relying only on short-term bills raises **rollover risk**.
- **GO vs revenue bonds:** GO = taxes of the local government; revenue = cash flows of a specific project, maturity matched to project life. Supranationals (World Bank, IMF, ADB) have the highest credit quality of these issuers.
- **External debt risk:** a US investor in USD bonds of an EM sovereign still has *indirect* currency exposure — the issuer must earn foreign currency to pay. [Note: The module text defines EM external debt as supranational debt plus foreign-currency bonds held by foreign investors, but an in-module question-set answer says many EM sovereigns also have external debt in their domestic currency.]

Module 6 — Fixed-Income Bond Valuation: Prices and Yields

Topic: Fixed Income · Exam relevance: **High-yield** · Calculation-heavy

The big picture

A bond's price is just the present value of its promised cash flows. Discount at a rate above the coupon and the bond sells at a **discount**; below the coupon and it sells at a **premium**. Between coupon dates the buyer also pays the seller the interest earned so far (accrued interest), which is why dealers quote a **flat** price but you pay the **full** price. When a bond does not trade, its yield is borrowed from similar bonds (**matrix pricing**).

Learning outcomes in plain language

- LOS 1 Price a bond from its YTM, on or between coupon dates.** Discount each cash flow at the periodic market discount rate; solve for YTM (the IRR) from a price; between coupon dates compute the full price, accrued interest (actual/actual vs 30/360) and flat price. YTM is realised only if the bond is held to maturity, there is no default and coupons are reinvested at the YTM.
- LOS 2 Relate price, coupon, maturity and YTM.** Price and yield move inversely; coupon < yield $\Rightarrow$ discount, = par, > premium; **coupon effect** (lower coupon, bigger % change); **maturity effect** (longer maturity, bigger % change — rare exceptions only for long, low-coupon discount bonds); **convexity** (price rise > price fall for equal yield moves); **pull to par** on the constant-yield price trajectory.
- LOS 3 Describe matrix pricing.** For illiquid or new bonds: find comparable bonds (similar maturity, coupon, credit quality), compute their YTMs and average by maturity, **linearly interpolate** to the target maturity, and discount the bond's cash flows at that yield. Also used to estimate a new issue's spread over the benchmark.

Key formulas

$$PV = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \dots + \frac{PMT_N + FV_N}{(1+r)^N} \quad PV^{Full} = PV^{Flat} + AI, \quad AI = \frac{t}{T} \times PMT$$

$$PV^{Full} = \frac{PMT}{(1+r)^{1-t/T}} + \frac{PMT}{(1+r)^{2-t/T}} + \dots + \frac{PMT + FV}{(1+r)^{N-t/T}} = PV \times (1+r)^{t/T}$$

where PMT = coupon per period; FV = face value; r = market discount rate (YTM) per period; N = periods to maturity; t = days from last coupon to settlement; T = days in the coupon period; PV = value as of the last coupon date (not the flat price). Zero-coupon: $PV = FV/(1+r)^N$. Interest rates are quoted annually: annual rate = rate per period $\times$ periods per year.

Day counts: actual/actual (government bonds) counts actual days; 30/360 (corporates) assumes 30-day months and a 360-day year. Accrued interest does not depend on the YTM, so only the flat price moves with rates. **Matrix pricing:** interpolated yield $= y_{short} + \frac{n - n_{short}}{n_{long} - n_{short}} (y_{long} - y_{short})$.

Worked example (CFA curriculum)

Par, discount, premium. BRWA 3.2% semiannual five-year bond (1.6 per period, $N = 10$): at 1.6% per period price = 100; at 2.0% price = **96.41**; at 1.2% price = **103.75** (−3.59% vs +3.75%: convexity).

Flat and full price, 90 days after the first coupon, $r = 2\%$ per period, 30/360 ($T = 180$): PV at last coupon ($N = 9$) = 96.735; $PV^{Full} = 96.735 \times 1.02^{90/180} = \mathbf{97.698}$; $AI = 90/180 \times 1.60 = 0.80$; $PV^{Flat} = 97.698 - 0.80 = \mathbf{96.898}$.

Matrix pricing (Bond X, 3-year, 4% semiannual). Two-year comparables yield 3.786% and 3.821% (average 3.8035%); five-year comparables 4.181% and 4.196% (average 4.1885%). Three-year yield = $3.8035\% + \frac{3-2}{5-2} (4.1885\% - 3.8035\%) = \mathbf{3.932\%}$.

$$\text{Price} = \sum_{t=1}^5 \frac{2}{1.019659^t} + \frac{102}{1.019659^6} = \mathbf{100.191}.$$

What examiners like to test

- **Flat vs full:** dealers quote the **flat** (clean) price; the buyer pays the **full** (dirty, invoice) price = flat + accrued interest. Quoting full prices would show a false daily rise and a drop on each coupon date.
- **Relationships:** for the same yield change, the zero or low-coupon bond and the longer bond move more (Romania 30-year: +100 bps gives −14.34% for the 4.625% coupon bond vs −24.83% for the zero). Discount and premium bonds with the same maturity **converge** to par.
- **Periodicity trap:** a 2% per half-year rate is a 4% annual yield. [Note: The text quotes the 1.2%-per-period BRWA price as 103.73 once; its own table and recomputation give 103.75. Its matrix-pricing formula line also prints 4.1855% for the 4.1885% five-year average.]

Module 7 — Yield and Yield Spread Measures for Fixed-Rate Bonds

Topic: Fixed Income · Exam relevance: **High-yield** · Calculation-heavy

The big picture

Two yields can only be compared if they are stated the same way: same **periodicity** (compounding frequency), same day count, and adjusted for any embedded option. Once they are comparable, a yield splits into a **benchmark** (top-down: expected real rate and inflation) and a **spread** (bottom-up: credit, liquidity, tax). Spreads are what analysts actually trade and track.

Learning outcomes in plain language

- LOS 1 Calculate annual yields for different compounding periods.** State YTM for a periodicity (rate per period $\times$ periods per year; semiannual bond basis = periodicity 2); convert between periodicities, including the effective annual rate (periodicity 1). More frequent compounding $\Leftrightarrow$ lower stated annual rate for the same return (also true for negative yields: more negative).
- LOS 2 Compare, calculate and interpret yield and spread measures.** Current yield; street convention vs true yield; government equivalent yield (30/360 restated to actual/actual); simple yield (JGBs); yield-to-call, **yield-to-worst**, option-adjusted yield; benchmark spread, **G-spread** (vs actual or interpolated government yield), **I-spread** (vs swap rate), **Z-spread** (constant spread over the spot curve), **OAS** = Z-spread less option value.

Key formulas

$$\left(1 + \frac{APR_m}{m}\right)^m = \left(1 + \frac{APR_n}{n}\right)^n \quad CY_t = \frac{\text{Annual coupon}_t}{\text{Bond price}_t} \quad \text{Yield}_{ACT/ACT} \approx \frac{365}{360} \times \text{Yield}_{30/360}$$

$$\text{Yield-to-call: } PV = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \dots + \frac{PMT + \text{Call price}}{(1+r)^N} \quad PV = \frac{PMT}{(1+z_1+Z)^1} + \dots + \frac{PMT + FV}{(1+z_N+Z)^N}$$

$$\text{OAS} = \text{Z-spread} - \text{Option value in bps per year} \quad \text{G-spread} = \text{YTM}_{\text{bond}} - \text{YTM}_{\text{government (actual or interpolated)}}$$

where APR_m = annual rate for periodicity m ; N = periods to the call date (yield-to-call) or maturity; $z_1 \dots z_N$ = benchmark spot rates; Z = Z-spread per period.
Yield-to-worst = lowest of all yields-to-call and the YTM. Option value (callable) = option-free price – callable price.

Worked example (CFA curriculum)

Periodicity. Antelas AG 3.20% quarterly bond at 94: YTM 4.548% (periodicity 4); BRWA 2.50% semiannual at 98.70: YTM 2.780% (periodicity 2). Convert Antelas: $(1 + 0.04548/4)^4 = (1 + APR_2/2)^2 \Rightarrow APR_2 = 4.574\%$. Extra yield = $4.574\% - 2.780\% = 179 \text{ bps}$, not 177 bps.

Yield-to-worst. VIVU 6.5% semiannual callable, price 106.50: yield-to-first call (103.25, $N = 6$) **5.149%**; second (102.50, $N = 8$) 5.247%; third (101.75, $N = 10$) 5.313%; fourth (101.00, $N = 12$) 5.362%; YTM ($N = 14$) 5.374%. Yield-to-worst = **5.149%**.

Interpolated G-spread. Russian Federation 5.25% semiannual, 24 years left, price 123.5: YTM = 3.756%. Treasuries: 20-year 2.00%, 30-year 2.25%; weights $w_{20} = (30 - 24)/(30 - 20) = 60\%$, $w_{30} = 40\%$: $r_{24y} = 0.6(2.00\%) + 0.4(2.25\%) = 2.10\%$. G-spread = $3.756\% - 2.10\% = 166 \text{ bps}$.

What examiners like to test

- **Compare like with like:** convert to a common periodicity (and day count) before subtracting yields; BRWA's 3.2% (30/360) is 3.244% actual/actual, so its spread over the 2.3% Treasury is 94.4 bps, not 90.
- **Which spread?** G-spread and I-spread use one yield for all cash flows; Z-spread uses the whole spot curve; OAS strips out the option. On-the-run government bonds trade at slightly lower yields than off-the-run. True yield is never above the street convention yield.
- **Option-adjusted price:** [Note: The text says the call option value is *added* to the callable bond's flat price to get the option-adjusted price, but a self-assessment answer says the option-adjusted price is *lower* than the flat price. Its Z-spread example (IIF bond) also has inconsistent benchmark-yield figures, so it is not used here.]

Module 8 — Yield and Yield Spread Measures for Floating-Rate Instruments

Topic: Fixed Income · Exam relevance: **High-yield** · Calculation-heavy

The big picture

A floater's coupon resets with the market reference rate (MRR), so its price stays near par when rates move; what moves its price is **credit**. If investors now demand a bigger spread (discount margin) than the fixed quoted margin, the floater trades at a discount. Money market instruments add a second twist: their rates are **simple, annualized, non-compounded** and quoted either as **discount rates** or **add-on rates** on 360- or 365-day years, so they must be restated before comparison.

Learning outcomes in plain language

- LOS 1 Calculate and interpret yield spread measures for floaters.** Coupon = MRR + **quoted margin** (QM, fixed at issue; set in advance, paid in arrears). **Required margin / discount margin** (DM) = spread that prices the FRN at par on a reset date. QM = DM $\Rightarrow$ par; DM > QM $\Rightarrow$ discount (PV of the “deficient” margin); DM < QM $\Rightarrow$ premium. Price an FRN, or solve for DM from its price, with the simplified model (same MRR for all periods, 30/360, on a reset date).
- LOS 2 Calculate and interpret money market yields.** Price and yield from a **discount rate** (CP, T-bills, bankers' acceptances; rate on face value, understates return) or an **add-on rate** (CDs, repos, MRRs; rate on amount invested); convert to a **bond equivalent yield** (365-day add-on rate) to compare; convert a money market rate (periodicity Year/Days) to a semiannual bond basis.

Key formulas

$$PV = \frac{\frac{(MRR + QM) \times FV}{m}}{\left(1 + \frac{MRR + DM}{m}\right)^1} + \dots + \frac{\frac{(MRR + QM) \times FV}{m} + FV}{\left(1 + \frac{MRR + DM}{m}\right)^N}$$

$$PV = FV \times \left(1 - \frac{\text{Days}}{\text{Year}} \times DR\right), \quad DR = \frac{\text{Year}}{\text{Days}} \times \frac{FV - PV}{FV}; \quad PV = \frac{FV}{1 + \frac{\text{Days}}{\text{Year}} \times AOR}, \quad AOR = \frac{\text{Year}}{\text{Days}} \times \frac{FV - PV}{PV}$$

where *MRR*, *QM*, *DM* = annual reference rate, quoted margin and discount margin; *m* = payments per year; *N* = periods to maturity; *DR* = discount rate; *AOR* = add-on rate; Days = days to maturity; Year = 360 or 365. **Bond equivalent yield** = AOR with Year = 365. Quoted “amount” = face value for discount-rate instruments, price at issuance for add-on instruments.

Worked example (CFA curriculum)

Discount margin. Antelas AG four-year FRN, three-month MRR = -0.55% (constant), QM = 250 bps, quarterly, price 97. $PMT = (-0.0055 + 0.0250) \times 100/4 = 0.4875$; with $PV = -97$, $FV = 100$, $N = 16$: $r = 0.686122\%$ per quarter. $DM = 4 \times 0.00686122 + 0.0055 = 3.29\%$ (329 bps > 250 bps, hence the discount). At a price of 102 the same steps give DM $\approx$ 199 bps (premium).

Discount-rate pricing. 91-day Indian T-bill, face INR10,000,000, DR = 3.45%, 360-day year: $PV = 10,000,000 \times (1 - \frac{91}{360} \times 0.0345) = \text{INR}9,912,791.67$; interest INR87,208.

Bond equivalent yields, 180 days. A: DR 4.33% (360) $\Rightarrow PV = 97.835$, BEY = $\frac{365}{180} \times \frac{2.165}{97.835} = 4.487\%$; B: DR 4.36% (365) $\Rightarrow 4.456\%$; C: AOR 4.35% (360) $\Rightarrow 4.410\%$; D: AOR 4.45% (365) = 4.450%. Highest return: **A**, even though its quoted rate is the lowest.

Periodicity. A 3.50% BEY on a 91-day T-bill (periodicity 365/91) equals **3.515%** on a semiannual bond basis.

What examiners like to test

- **Quoted vs required margin:** a downgrade raises the required (discount) margin, not the coupon spread. Floater interest rate sensitivity comes from the time to the next reset: longer reset periods mean more price volatility.
- **Discount rate understates yield** because it divides interest by FV, not PV. Always convert every instrument to BEY (365-day add-on) before ranking.
- **Book slip:** [Note: In the curriculum CP-vs-CD example, 90-day CP at a 0.100% discount rate converts to a 0.1014% BEY, yet the text concludes the CP beats a 0.120% CD by 0.2 bps; on its own numbers the CD yields about 1.9 bps more.]

Module 9 — The Term Structure of Interest Rates: Spot, Par, and Forward Curves

Topic: Fixed Income · Exam relevance: **High-yield** · Calculation-heavy

The big picture

Interest rates differ by maturity, so a single YTM is only an average. **Spot rates** (zero-coupon, default-risk-free yields) give each cash flow its own discount rate and produce the **no-arbitrage** price. From the spot curve come two more curves: the **par curve** (coupons that would price bonds at 100) and the **forward curve** (breakeven reinvestment rates for future periods). All three carry the same information.

Learning outcomes in plain language

- LOS 1 Define spot rates and the spot curve; price a bond with spot rates.** Spot (zero) rate = YTM on a default-risk-free zero-coupon bond; the spot curve plots them by maturity (in practice built from recent coupon-paying government bonds plus interpolation). Discount each cash flow at its own spot rate; add a spread for riskier bonds.
- LOS 2 Define par and forward rates and calculate them.** **Par rate** = coupon/YTM that prices a bond at par given the spot curve. **Implied forward rate** “AyBy” (e.g. 3y1y) = breakeven reinvestment rate linking a shorter and a longer zero. Compute par rates from spot rates, forward rates from spot rates, spot rates from forward rates (geometric average), and price a bond with forward rates.
- LOS 3 Compare the spot, par and forward curves.** Upward-sloping spot curve: par curve slightly **below**, forward curve **above**. Flat: all three equal. Inverted: par **above**, forward **below**. The gap between spot and par grows at longer maturities.

Key formulas

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT+FV}{(1+Z_N)^N} \quad 100 = \frac{PMT}{(1+z_1)^1} + \frac{PMT}{(1+z_2)^2} + \cdots + \frac{PMT+100}{(1+z_N)^N}$$

$$(1+Z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+Z_B)^B \quad (1+Z_N)^N = (1+0y1y)(1+1y1y) \cdots (1+(N-1)y1y)$$

where Z_n = spot rate for period n ; PMT = coupon; solving the second equation for PMT gives the par rate ($PMT/100$); $IFR_{A,B-A}$ = implied forward rate starting at time A with tenor $B-A$ (“Ay(B-A)y”). Rates are effective annual (periodicity 1) in the curriculum examples.

Interpretation: if you expect the future one-year rate to exceed the implied forward rate, buy the shorter bond and reinvest; if below, buy the longer bond.

Worked example (CFA curriculum)

Spot-rate pricing. Spot rates 2%, 3%, 4%; 3-year 5% annual bond: $\frac{5}{1.02} + \frac{5}{1.03^2} + \frac{105}{1.04^3} = 4.902 + 4.713 + 93.345 = 102.960$. Its YTM is **3.935%** (same price, different PV of each cash flow).

Par rates. Spot rates 5.263%, 5.616%, 6.359%, 7.008% give par rates **5.263%, 5.606%, 6.306%, 6.899%** — each slightly below its spot rate (upward-sloping curve). Check: $5.606/1.05606 + 105.606/1.05606^2 = 100$.

Forward from spot. 3-year zero 3.65%, 4-year zero 4.18%: $(1.0365)^3(1+IFR_{3,1}) = (1.0418)^4 \Rightarrow 3y1y = 5.79\%$.

Spot from forward, and pricing. 0y1y 1.88%, 1y1y 2.77%, 2y1y 3.54%, 3y1y 4.12% give spot rates 1.88%, **2.324%, 2.728%, 3.074%**. A 4-year 3.75% annual bond priced with the forward rates:

$$\frac{3.75}{1.0188} + \frac{3.75}{1.0188 \times 1.0277} + \frac{3.75}{1.0188 \times 1.0277 \times 1.0354} + \frac{103.75}{1.0188 \times 1.0277 \times 1.0354 \times 1.0412} = 102.637,$$

identical to discounting at the implied spot rates: $\frac{3.75}{1.0188} + \frac{3.75}{1.02324^2} + \frac{3.75}{1.02728^3} + \frac{103.75}{1.030741^4} = 102.637$.

What examiners like to test

- **Naming:** “2y3y” starts in 2 years and lasts 3 years; it needs the 2-year and 5-year spot rates. Spot rates are geometric (not arithmetic) averages of the forwards.
- **Curve shapes:** with an upward-sloping spot curve forwards are above spot and par is just below spot; reverse for an inverted curve. Negative spot rates can still produce positive forwards if the curve slopes up.
- **Same price, different curves:** two very different spot curves can price a bond identically if the relevant spot rates (e.g. the 3-year) are almost the same. [Note: Some in-module worked solutions contain misprints (e.g. the two-year spot rate equation prints exponents 2 and 3); the figures above use the main-text versions, all recomputed.]

Module 10 — Interest Rate Risk and Return

Topic: Fixed Income · Exam relevance: **High-yield** · Calculation + concept

The big picture

A bond investor earns coupons, interest on reinvested coupons, and a gain or loss if the bond is sold before maturity. When rates rise, reinvestment income goes up but the sale price goes down; when rates fall, the opposite. Which effect wins depends on the **investment horizon**. At the horizon equal to the bond's **Macaulay duration** the two effects roughly cancel, so the investor is (nearly) immunised against a one-time parallel rate shift.

Learning outcomes in plain language

- LOS 1 Calculate and interpret the sources of return from a fixed-rate bond.** (1) promised coupons and principal, (2) reinvestment of coupons (interest-on-interest), (3) capital gain or loss if sold before maturity versus the **carrying value** on the constant-yield price trajectory. Compute the **horizon yield** (annualised holding-period return). Horizon yield = original YTM only if coupons are reinvested at that YTM and the bond is sold on the constant-yield trajectory.
- LOS 2 Relate holding period return, Macaulay duration and investment horizon.** Buy-and-hold (horizon = maturity): only reinvestment risk. Short horizon: mainly price risk. Horizon > MacDur: reinvestment risk dominates, fear **falling** rates; horizon < MacDur: price risk dominates, fear **rising** rates; horizon = MacDur: the two offset. Duration gap = MacDur – horizon.
- LOS 3 Define, calculate and interpret Macaulay duration.** PV-weighted average time to receipt of the cash flows (weights = each cash flow's share of the full price); less than maturity for a coupon bond, equal to maturity for a zero (on a coupon date); falls as time passes; divide by periods per year to annualise.

Key formulas

$$r = \left(\frac{FV}{PV} \right)^{1/T} - 1 \quad \text{Duration gap} = \text{Macaulay duration} - \text{Investment horizon}$$

$$\text{MacDur} = \left\{ (1 - t/T) \left[\frac{\frac{PMT}{(1+r)^{1-t/T}}}{PV^{Full}} \right] + (2 - t/T) \left[\frac{\frac{PMT}{(1+r)^{2-t/T}}}{PV^{Full}} \right] + \dots + (N - t/T) \left[\frac{\frac{PMT + FV}{(1+r)^{N-t/T}}}{PV^{Full}} \right] \right\}$$

$$\text{MacDur} = \left\{ \frac{1+r}{r} - \frac{1+r + [N \times (c-r)]}{c \times [(1+r)^N - 1] + r} \right\} - \frac{t}{T}$$

where FV (return formula) = reinvested coupons + sale price or redemption; PV = purchase price; T (return formula) = years held; r = YTM per period; c = coupon rate per period; N = periods to maturity from the start of the current period; t = days since the last coupon; T (duration) = days in the coupon period.

Worked example (CFA curriculum)

BRWA 10-year, 6.2% annual eurobond bought at par (YTM 6.20%); Macaulay duration at issuance = **7.7429** years (final cash flow 106.2 is 58% of PV). Rates move once, immediately after purchase:

Horizon yield	VFO (10 years, hold)	Highest (8 years)	Baywhite (4 years)
Rates stay 6.20%	6.20%	6.20%	6.20%
Rise to 7.20%	6.43% (FV coupons 86.475)	6.24% (64.071 + 98.197)	5.28% (27.609 + 95.263)
Fall to 5.20%	5.98% (78.715)	6.17% (59.630 + 101.854)	7.16% (26.802 + 105.043)

E.g. Baywhite, rates up: reinvested coupons = $6.2 \times \frac{1.072^4 - 1}{0.072} = 27.609$; sale price with 6 years left at 7.2% = 95.263; $r = \left(\frac{27.609 + 95.263}{100} \right)^{1/4} - 1 = \mathbf{5.28\%}$. Highest's 8-year horizon is close to the 7.74-year duration, so its return barely moves; 57 days after issue the duration has already fallen to 7.5845.

What examiners like to test

- **Sign of the duration gap:** positive gap (MacDur > horizon) $\Rightarrow$ rising rates hurt (price risk); negative gap $\Rightarrow$ falling rates hurt (reinvestment risk). A buy-and-hold investor faces *only* reinvestment risk.
- **Semiannual bonds:** Macaulay duration from the table is in periods — divide by 2 for years (e.g. 5.713 periods = 2.857 years for a 3-year 4% semiannual par bond).
- **Zero-coupon bond:** Macaulay duration = time to maturity (on a coupon date), even with a negative yield.

Module 11 — Yield-Based Bond Duration Measures and Properties

Topic: Fixed Income · Exam relevance: **High-yield** · Calculation-heavy

The big picture

Price and yield move in opposite directions, but not by the same amount for every bond. **Modified duration** is the slope of the price–yield curve expressed as a percentage: it turns a change in yield into an estimated percentage change in the **full** price. Money duration and PVBP restate the same slope in currency. All of these are **yield durations**: they assume the cash flows are certain and that only the bond's own YTM changes.

Learning outcomes in plain language

- LOS 1 Define, calculate and interpret ModDur, money duration and PVBP.** Compute modified duration from Macaulay duration (dividing by one plus the yield *per period*, then annualising) or approximate it by repricing the bond at yields $\pm \Delta \text{Yield}$; use it to estimate $\% \Delta PV^{Full}$. Scale by the full price or position value to get **money duration** (the currency change); compute the **PVBP** (PV01, DV01) from prices at ± 1 bp. Know the special cases: zero-coupon (MacDur = maturity), perpetuity and floating-rate note.
- LOS 2 Explain how maturity, coupon and yield affect interest rate risk.** All else equal, duration is **higher** for a longer time-to-maturity, a **lower** coupon and a **lower** YTM, and falls as the coupon period elapses (“saw-tooth” pattern). Recognise the exception: a long-dated **discount** bond's MacDur can exceed $(1+r)/r$, peak and then fall as maturity lengthens.

Key formulas

$$\begin{aligned} \text{ModDur} &= \frac{\text{MacDur}}{(1+r)}, & \% \Delta PV^{Full} &\approx -\text{AnnModDur} \times \Delta \text{AnnYield}, & \text{AnnModDur} &\approx \frac{(PV_-) - (PV_+)}{2 \times (\Delta \text{Yield}) \times (PV_0)} \\ \text{MoneyDur} &= \text{AnnModDur} \times PV^{Full}, & \Delta PV^{Full} &\approx -\text{MoneyDur} \times \Delta \text{Yield}, & \text{PVBP} &= \frac{(PV_-) - (PV_+)}{2} \\ \text{AnnMacDur} &\approx \text{AnnModDur} \times (1+r), & \text{MacDur}_{\text{perpetuity}} &= \frac{1+r}{r}, & \text{MacDur}_{\text{floating}} &= \frac{(T-t)}{T} \\ \text{MacDur} &= \left\{ \frac{1+r}{r} - \frac{1+r + [N \times (c-r)]}{c \times [(1+r)^N - 1] + r} \right\} - \frac{t}{T} \end{aligned}$$

where r = yield per period; c = coupon rate per period; N = number of periods; t/T = fraction of the coupon period elapsed; PV_0 = full price; PV_+ , PV_- = full prices after raising/lowering the YTM by ΔYield . ModDur is annualised by dividing by periods per year. Percentage changes apply to the **full** (dirty) price. A related “basis point value” (BPV) is $\text{MoneyDur} \times 0.0001$.

Worked example (CFA curriculum)

BRWA 5-year, 3.2% semiannual bond priced at par (15 Oct 2025 to 15 Oct 2030). MacDur at issuance = 9.3203 semiannual periods, so $\text{ModDur} = 9.3203 / (1 + 0.032/2) = 9.17351$ and **AnnModDur** = $9.17351 / 2 = \mathbf{4.58676}$. A +80 bp instant rise in yield: $\% \Delta PV^{Full} \approx -4.58676 \times 0.0080 = \mathbf{-3.67\%}$ (and +3.67% for –80 bp: duration alone is symmetric).

Approximation with ± 5 bp: $PV_+ = 99.771$, $PV_- = 100.230$, so $\text{AnnModDur} \approx (100.230 - 99.771) / (2 \times 0.0005 \times 100) = \mathbf{4.587}$.

Money duration and PVBP (settlement 57 days into the period, full price 100.504, AnnModDur 4.43092). A USD100,000,000 par position has market value USD100,503,921 and $\text{MoneyDur} = \text{USD}445,324,719$; a +100 bp move loses about **USD4,453,247**. Repricing at 3.21% and 3.19% gives $PV_+ = 100.459400$, $PV_- = 100.548465$, so **PVBP** = $(100.548465 - 100.459400) / 2 = \mathbf{0.044532}$ per 100 par.

Special cases. Perpetuity yielding 6%: $\text{MacDur} = 1.06 / 0.06 = 17.667$ versus 5.0 for a 5-year zero. FRN 57 days into a 180-day reset period: $\text{MacDur} = (180 - 57) / 180 = 0.683333$.

What examiners like to test

- **Per-period yield:** divide MacDur by $1 + r$ using the *periodic* yield (1.016 for 3.2% semiannual), then annualise. With a **negative** yield, ModDur exceeds MacDur.
- **Ranking risk:** lower coupon, lower YTM or longer maturity means higher duration; a premium bond has a lower duration than a discount bond of the same maturity and YTM. FRNs have very low duration (at most one reset period).
- **Linear approximation:** duration gives equal $\pm$ estimates, but the true price–yield curve is convex, so the actual gain for a yield fall exceeds the loss for an equal yield rise (added in LM 12).

Module 12 — Yield-Based Bond Convexity and Portfolio Properties

Topic: Fixed Income · Exam relevance: **High-yield** · Calculation-heavy

The big picture

Duration draws a straight tangent line through a curved price–yield relationship, so it overstates losses when yields rise and understates gains when yields fall. **Convexity** is the second-order correction: for an option-free bond it is always positive, so adding it always *raises* the estimated price. The same duration and convexity statistics can be averaged across bonds to describe a whole portfolio, at the cost of assuming parallel yield-curve shifts.

Learning outcomes in plain language

- LOS 1 Calculate and interpret convexity; describe the convexity adjustment.** Compute annual convexity from a cash-flow table (sum of the convexity column divided by periods per year squared) or approximate it from PV_+ , PV_- , PV_0 . The adjustment $\frac{1}{2} \times \text{AnnConvexity} \times (\Delta \text{Yield})^2$ is always positive for option-free bonds. Convexity rises with longer maturity, lower coupon, lower YTM and greater dispersion of cash flows; investors value (and pay for) convexity.
- LOS 2 Estimate the percentage price change using duration and convexity.** Combine the duration effect and the convexity adjustment for a given yield change; in currency terms, use money duration and **money convexity** ($\text{AnnConvexity} \times \text{full price}$). The adjustment matters most for large yield changes and long, low-coupon bonds.
- LOS 3 Calculate portfolio duration and convexity and explain limitations.** Two methods: (1) aggregate cash flows of the whole portfolio (theoretically correct, hard in practice); (2) market-value-weighted averages of each bond's duration and convexity (common in practice). Method 2 implicitly assumes a **parallel shift** (all yields change by the same amount) and is more accurate when the bonds' yields are similar and the curve is flat.

Key formulas

$$\% \Delta PV^{Full} \approx (-\text{AnnModDur} \times \Delta \text{Yield}) + \left[\frac{1}{2} \times \text{AnnConvexity} \times (\Delta \text{Yield})^2 \right], \quad \text{ApproxCon} = \frac{(PV_-) + (PV_+) - [2 \times (PV_0)]}{(\Delta \text{Yield})^2 \times (PV_0)}$$

$$\text{MoneyCon} = \text{AnnConvexity} \times PV^{Full}, \quad \Delta PV^{Full} \approx -(\text{MoneyDur} \times \Delta \text{Yield}) + \left[\frac{1}{2} \times \text{MoneyCon} \times (\Delta \text{Yield})^2 \right]$$

$$\text{Convexity of each cash flow (per period)} = t \times (t + 1) \times w_t \times (1 + r)^{-2}; \quad \text{AnnConvexity} = \frac{\sum(\text{column})}{(\text{periods per year})^2}$$

$$\text{Portfolio duration} \approx \sum_i w_i \times \text{AnnModDur}_i, \quad \text{Portfolio convexity} \approx \sum_i w_i \times \text{AnnConvexity}_i$$

where t = time to receipt in periods; w_t = PV of the cash flow as a share of the full price; r = periodic yield; w_i = bond i 's share of portfolio **market** value. Convexity must be scaled to the same order of magnitude as duration squared, with ΔYield in decimals.

Worked example (CFA curriculum)

BRWA 5-year, 3.2% semiannual bond at par. The convexity column sums to 96.9558, so $\text{AnnConvexity} = 96.9558/2^2 = 24.2389$. With ± 5 bp prices 99.770965 and 100.229641: $\text{ApproxCon} = (100.229641 + 99.770965 - 200)/(0.0005^2 \times 100) = 24.23896$.

± 100 bp estimate (AnnModDur 4.58676): $-4.58676 \times 0.01 + \frac{1}{2} \times 24.23895 \times 0.01^2 = -4.46556\%$ and $+4.58676\% + 0.12119\% = +4.70795\%$. Actual repricing: -4.46788% and $+4.71035\%$. Duration alone misses by about 12 bp; with convexity the error is under $\frac{1}{4}$ bp. For a USD100 million position the +100 bp estimate is $-\text{USD}458,675,875 \times 0.01 + \frac{1}{2} \times 2,423,894,503 \times 0.01^2 = -\text{USD}4,465,564$ versus an actual $-\text{USD}4,467,884$.

Portfolio. USD50 million par each of BRWA (price 100.000, dur 4.58676, conv 24.23896) and the 30-year Romania bond (price 98.022, dur 15.90637, conv 369.64203); weights 0.5050 / 0.4950. Portfolio duration = **10.19004**, convexity = **195.21581**; for +100 bp: $-10.19004 \times 0.01 + \frac{1}{2} \times 195.21581 \times 0.01^2 = -9.21396\%$. [Note: the book's Example 1 prints the Romanian ApproxCon as 369.58881, but its comparison table and Exhibit 6 use 369.64203; recomputing with unrounded prices gives 369.64203]

What examiners like to test

- **Sign of the convexity term:** $(\Delta \text{Yield})^2$ is positive whether yields rise or fall, so for option-free bonds the adjustment is always *added*. A common error is subtracting it for a yield decrease.
- **Scaling:** a reported convexity of 0.235 with duration 5.0 must be rescaled to 23.5 before use; forgetting to divide the cash-flow sum by $(\text{periods per year})^2$ gives a convexity four times too big for a semiannual bond.
- **Same duration, more convexity:** the more convex bond outperforms whether rates rise or fall (if convexity is not priced in). Weighted-average measures assume parallel shifts, so they miss steepening, flattening and twists.

Module 13 — Curve-Based and Empirical Fixed-Income Risk Measures

Topic: Fixed Income · Exam relevance: High-yield · Calculation + concept

The big picture

When a bond has an embedded option its cash flows depend on future rates, so it has no well-defined YTM and yield duration breaks down. Instead we shift the **benchmark curve** and reprice the bond with an option model: that gives **effective** (curve) duration and convexity. Key rate durations split that sensitivity by maturity, and **empirical** duration asks what history says actually happens when benchmark yields and credit spreads move together.

Learning outcomes in plain language

- LOS 1 Why effective duration and convexity suit bonds with embedded options.** Callable/puttable bonds and MBS lack a well-defined YTM, so MacDur and ModDur are inappropriate; effective measures use prices from an option pricing model after parallel shifts of the benchmark (government par) curve. A call **lowers** effective duration when rates fall and can make effective convexity **negative**; a put lowers effective duration when rates rise, and puttable bonds **always** have positive convexity. For option-free bonds EffDur and ModDur differ slightly unless the curve is flat.
- LOS 2 Estimate the percentage price change from effective duration and convexity.** Apply the duration-plus-convexity formula with ΔCurve in place of ΔYield ; with negative convexity the adjustment *reduces* the estimate in both directions. A smaller ΔCurve does not necessarily improve accuracy for complex securities.
- LOS 3 Define key rate duration and its use.** Key rate (partial) duration = sensitivity to a shift in the benchmark yield at one maturity, holding others fixed; key rate durations sum to effective duration. They measure **shaping risk** (steepening, flattening, twists) and let managers tilt exposures by tenor.
- LOS 4 Contrast empirical and analytical duration.** Analytical duration comes from mathematical formulas and assumes benchmark yields and spreads are uncorrelated; empirical duration is estimated from historical data in statistical models. In a flight to quality, benchmark yields fall while credit spreads widen, so for credit-risky bonds **empirical duration** < **analytical duration**; for government bonds the two are similar.

Key formulas

$$\text{EffDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Curve}) \times (PV_0)}, \quad \text{EffCon} = \frac{[(PV_-) + (PV_+)] - [2 \times (PV_0)]}{(\Delta\text{Curve})^2 \times (PV_0)}$$

$$\% \Delta PV^{\text{Full}} \approx (-\text{EffDur} \times \Delta\text{Curve}) + \left[\frac{1}{2} \times \text{EffCon} \times (\Delta\text{Curve})^2 \right]$$

$$\text{KeyRateDur}_k = -\frac{1}{PV} \times \frac{\Delta PV}{\Delta r_k}, \quad \sum_{k=1}^n \text{KeyRateDur}_k = \text{EffDur}, \quad \frac{\Delta PV}{PV} = -\text{KeyRateDur}_k \times \Delta r_k$$

where PV_+ , PV_- = full prices from the option pricing model after the benchmark par curve is shifted up/down by ΔCurve , holding constant call protection, call schedule, credit spread and rate volatility; r_k = the k th key rate (e.g., 0.5, 2, 5, 10, 20, 30 years).

Worked example (CFA curriculum)

Callable bond. $PV_0 = 101.060$; curve ± 25 bp gives $PV_+ = 99.050$, $PV_- = 102.891$. $\text{EffDur} = (102.891 - 99.050) / (2 \times 0.0025 \times 101.060) = 7.601$; $\text{EffCon} = (102.891 + 99.050 - 202.120) / (0.0025^2 \times 101.060) = -283$. Duration meets a 7–8 target, but convexity is negative, so the analyst advises against buying. For ± 100 bp (book uses $\text{EffCon} -285.168$): $-7.601\% - 1.43\% = -9.03\%$ and $+7.601\% - 1.43\% = +6.17\%$: losses exceed gains. [Note: Example 1 computes EffCon as -283 but Example 2 uses -285.168 ; with -283.4 the results are $-9.02\% / +6.18\%$. Similarly the BRWA illustration gets EffCon 40.000 from its prices but later uses 26.723]

Key rate durations. Bonds A–D (1, 5, 10, 20 years) have key rate durations 0.645, 1.483, 2.158, 2.982; forecast par-curve changes -23 , -25 , -18 , -10 bp. Price changes: $-0.645 \times -0.0023 = +0.148\%$, then $+0.371\%$, $+0.388\%$, $+0.298\%$, so overweight B and C, funded mostly from A.

What examiners like to test

- **Negative convexity:** for a callable bond, the price rise from a rate fall is capped near the call price; with $\text{EffCon} < 0$ the convexity term is *subtracted* for both up and down moves. Puttable bonds never have negative convexity.
- **Which duration?** Bonds with embedded options or MBS: effective (curve) duration. Non-parallel shifts: key rate durations. Credit-risky bonds in stress: empirical duration.
- **Flight to quality:** a fall in government yields is partly offset by wider spreads, so a high-yield bond's price rises less than analytical duration predicts. The analytical duration itself does **not** change.

Module 14 — Credit Risk

Topic: Fixed Income · Exam relevance: **High-yield** · Calculation + concept

The big picture

Credit risk is the risk of loss because a borrower does not pay interest and principal in full and on time. It has two parts: **how likely** default is (POD) and **how much** is lost if it happens (LGD). Investors are paid for bearing it through the **credit spread**; ratings summarise it, and spreads move with the economy, market liquidity and issuer news, changing bond prices through duration and convexity.

Learning outcomes in plain language

- LOS 1 Describe credit risk and its components, POD and LGD.** Expected loss = $\text{POD} \times \text{LGD}$, where $\text{LGD} = \text{expected exposure} \times (1 - \text{recovery rate})$. The credit spread approximates $\text{POD} \times \text{LGD}$, so compare them to judge compensation. POD is driven by profitability, coverage and leverage; LGD mainly by seniority and security. Know the “Cs” of credit (capacity, capital, collateral, covenants, character; conditions, country, currency) and the illiquid vs insolvent distinction.
- LOS 2 Describe uses and limitations of credit ratings.** Moody’s/S&P/Fitch give symbol-based, forward-looking ratings paid for by the **issuer**; investment grade = Baa3/BBB– or higher, high yield = Ba1/BB+ or lower. Ratings allow comparison and feed regulation and covenants. Pitfalls: ratings are sticky and **lag** market prices; some risks are hard to capture (litigation, environmental, debt-financed buybacks, split ratings); and they can be wrong (subprime MBS, Enron, Wirecard).
- LOS 3 Describe factors that influence the level and volatility of yield spreads.** *Macro*: spreads narrowest near the top of the credit cycle and widest near the bottom; HY spreads move more than IG. *Market*: liquidity (bid–offer), dealer capital, supply and demand; flight to quality. *Issuer*: coverage and leverage. Price impact of a spread change uses the same duration and convexity as a yield change.

Key formulas

$$\text{EL} = \text{POD} \times \text{LGD}, \quad \text{LGD} = \text{EE} \times (1 - \text{RR}), \quad \text{Credit Spread} \approx \text{POD} \times \text{LGD}$$

$$\% \Delta PV^{Full} = -\text{AnnModDur} \times \Delta \text{Spread} \quad (\text{small}); \quad \% \Delta PV^{Full} = -(\text{AnnModDur} \times \Delta \text{Spread}) + \frac{1}{2} \text{AnnConvexity} \times (\Delta \text{Spread})^2$$

where POD = probability of default (usually annual); EE = expected exposure (exposure at default, e.g., face value plus accrued interest less collateral value); RR = recovery rate; LGD can be in currency or as % of principal. Yield spread = liquidity spread (yield at bid minus yield at offer) + credit spread.

Moody’s	S&P / Fitch	Grade	Meaning
Aaa – Baa3	AAA – BBB–	Investment grade	Highest to moderate credit quality; lower spreads, fewer investor restrictions
Ba1 – B3	BB+ – B–	High yield (speculative)	Substantial to high credit risk
Caa1 – C	CCC+ – C / D	Distressed / default	Default a real possibility or already occurring (D = in default for S&P and Fitch)

Worked example (CFA curriculum)

Expected loss vs spread. BRWA 5-year bond: 3.2% yield vs 2.3% Treasury, G-spread 90 bp. POD 1%, LGD 80%: $\text{EL} = 0.01 \times 80\% = 0.80\% < 0.90\%$ spread, so investors are compensated. VIVU (treated as 5-year non-callable): $6.5\% - 2.3\% = 420$ bp spread; POD 6%, LGD 50%: $\text{EL} = 3.0\% < 4.2\%$.

Currency EL. Exposure EUR400,000, POD 5%, RR 90%: $\text{EL} = 0.05 \times 400,000 \times (1 - 0.9) = \text{EUR}2,000$.

Downgrade. A one-notch downgrade widens BRWA spreads by 1.0% (2030 bond: ModDur 3.82250, convexity 20.22640) and 2.2% (2035 bond: 6.57882, 58.43082). 2030s: $-(3.82250 \times 0.01) + \frac{1}{2}(20.2264)(0.01)^2 = -3.7214\%$; 2035s: $-(6.57882 \times 0.022) + \frac{1}{2}(58.43082)(0.022)^2 = -13.0594\%$.

Spread decomposition (Romania 2049 Eurobond). Mid-price yield 3.2988% – Bund 0.2350% = 306.38 bp; liquidity spread $3.3588\% - 3.2396\% = 11.92$ bp; credit spread = **294.46** bp. [Note: the book quotes bid/offer 122.25/125.75 but computes the yields at 122.75 and 125.25; the 11.92 bp uses the latter]

What examiners like to test

- **EL arithmetic:** multiply by $(1 - \text{RR})$, not RR; do not forget to reduce exposure for collateral. Spread $> \text{POD} \times \text{LGD}$ means the investor is (more than) fairly compensated.
- **Ratings are issuer-paid and lag prices;** an outlook moves before a rating. Two bonds with the same rating can trade at very different spreads, especially in distress (pricing then focuses on default timing and recovery).
- **Longer duration = more spread sensitivity:** the same spread change moves a long bond far more; reported convexity (e.g., 0.23) must be rescaled (to 23) before use.

Module 15 — Credit Analysis for Government Issuers

Topic: Fixed Income · Exam relevance: **Medium-yield** · **Concept-heavy**

The big picture

Governments borrow to run fiscal policy and provide public goods, and they repay from **taxes** and other government revenue rather than operating cash flow. A sovereign's power to tax everything in its jurisdiction usually makes it the lowest credit risk in that country, but investors cannot force a sovereign into bankruptcy, so its **willingness** to pay matters as much as its ability. Non-sovereign issuers borrow on their own taxing power or on the revenue of a specific project.

Learning outcomes in plain language

LOS 1 Explain special considerations when evaluating sovereign and non-sovereign government credit. *Sovereigns:* combine qualitative factors (institutions and policy, including willingness to pay under **sovereign immunity**; fiscal flexibility; monetary effectiveness and central bank independence; economic flexibility and diversification; external status, especially **reserve currency** status) with quantitative factors (fiscal strength, economic growth and stability, external stability), using government revenue or GDP as the denominator instead of sales or assets. *Non-sovereigns:* agencies, government-sector banks and development institutions usually get the sovereign's rating through implicit or explicit support; supranationals are owned by several sovereigns; regional governments issue **general obligation** bonds (backed by taxing power, analysed like a sovereign but without monetary or full fiscal powers) or **revenue bonds** (backed by one project, analysed like a corporate using the **debt service coverage ratio**), which are riskier than GO bonds.

Key facts

Qualitative factors (sovereign)

Institutions & policy	Rule of law, property rights, transparency, political stability, willingness to pay ; restructurings (e.g., Uruguay 2003 exchange) can be defaults in substance
Fiscal flexibility	Ability to adjust revenue and spending, fiscal discipline, prudent use of debt
Monetary effectiveness	Central bank independence (less risk of monetising debt), policy credibility, exchange rate regime, developed financial system
Economic flexibility	Size, per capita income, diversification, competitiveness, adaptability to shocks
External status	Reserve currency status, access to external funding, geopolitical risk

Quantitative factors (higher debt ratios = lower credit quality)

Fiscal strength	Debt burden: debt/GDP, debt/revenue. Debt affordability: interest/revenue, interest/GDP. Also budget surplus or deficit/GDP
Growth & stability	GDP (PPP) size, per capita GDP, average real GDP growth and its volatility
External stability	FX reserves/GDP, reserve ratio (FX reserves/external debt), LT external debt/GDP, external debt due in 12 months/GDP

Non-sovereign issuers

Agencies, public banks	Rated like the sovereign (e.g., KfW: explicit statutory guarantee, AAA)
Supranationals	Owned by several member sovereigns (World Bank/IBRD, ADB)
GO bonds	Unsecured, backed by general taxing power; tax base, local economy, budget management
Revenue bonds	One project (toll road, metro, hospital); project cash flows, DSCR, coverage covenants

Worked example (CFA curriculum)

External stability in Southeast Asia (FYE 2020). Reserve ratio / FX reserves to GDP: Malaysia 54% / 11.9%; Thailand 102% / 16.4%; Indonesia 31% / 3.9%; Vietnam 84% / 9.9%; Philippines 96% / 10.3%. The ratios alone suggest Thailand is strongest, yet Moody's rates Malaysia higher (A3 vs Baa1) for its competitiveness and lower foreign currency debt, and rates Thailand the same as the Philippines, whose remittances from workers abroad were nearly 10% of GDP (under 2% in Thailand). Lesson: ratios are a starting point, not the answer.

Which sovereign rates lowest? LT external debt/GDP: Costa Rica 39.6%, Dominican Republic 35.7%, El Salvador 57.0%; GDP growth 2.06%, 5.07%, 2.39%; reserves/GDP 13.9%, 8.1%, 12.4%. **El Salvador:** by far the heaviest external debt burden, and only modest growth.

What examiners like to test

- **Classify the ratio:** debt/GDP and interest/revenue are *fiscal strength*; FX reserves/GDP is *external stability*; GDP/population is *growth and stability*. Debt burden resembles leverage (solvency), debt affordability resembles coverage.
- **GO vs revenue vs agency:** a bond backed by a region's taxes is GO even if proceeds go to a green project; a bond repaid from a metro line's revenue is a revenue bond even with a sovereign guarantee. Local governments are usually rated at or below the sovereign. [India context, not curriculum] Indian state and municipal bonds are sub-sovereign debt: classify them by their source of repayment.
- **Non-reserve currency sovereigns:** what matters is the ability to generate stable foreign currency inflows; capital controls and fixed exchange rates are not strengths.

Module 16 — Credit Analysis for Corporate Issuers

Topic: Fixed Income · Exam relevance: **Medium-yield · Calculation + concept**

The big picture

A company repays debt from the cash its business generates, so corporate credit analysis asks two questions. How likely is default? That depends on the business model, industry, governance, and on profitability, leverage, coverage and liquidity, measured with ratios. How much would a given bond lose? That depends on where the bond sits in the **priority of claims**, which is why one issuer's bonds can carry different ratings.

Learning outcomes in plain language

- LOS 1 Describe qualitative and quantitative factors in corporate creditworthiness.** *Qualitative:* business model (stability and predictability of revenue and margins, asset quality), industry and competition (Porter-type forces, cyclicality, life cycle), business risk (issuer-specific, industry and external risks including ESG) and **corporate governance** (use of proceeds, treatment of debtholders, aggressive accounting, frequent auditor or CFO changes). Longer-dated debt is more exposed to long-term strategy risk. Secured lenders prefer tangible (hard) collateral. *Quantitative:* forecasts (top-down, bottom-up or hybrid) of **profitability, leverage, coverage and liquidity**.
- LOS 2 Calculate and interpret credit ratios.** Higher EBIT margin, higher coverage and lower leverage mean lower credit risk. Use cash-flow measures (FFO, RCF, FCF), adjust debt for leases and off-balance-sheet items, compare with peers and rating bands, and project ratios under stress scenarios. Ratio levels for a rating differ widely by industry.
- LOS 3 Describe seniority, secured vs unsecured debt, priority of claims and effect on ratings.** Ranking: first lien/mortgage, senior secured, junior secured, senior unsecured, senior subordinated, subordinated, junior subordinated, then equity. Secured claims beyond collateral value become senior unsecured. Same-seniority debt ranks **pari passu** regardless of maturity. Recovery varies by seniority, industry and point in the cycle; bankruptcy negotiations can depart from strict priority. **Issuer** rating = senior unsecured; **issue** ratings are **notched** for LGD, more so for lower-rated issuers; holding-company debt is **structurally subordinated**.

Key formulas

$$\text{EBIT margin} = \frac{\text{Operating income}}{\text{Revenue}}, \quad \text{EBIT to interest} = \frac{\text{Operating income}}{\text{Interest expense}}, \quad \text{Debt to EBITDA} = \frac{\text{Debt}}{\text{EBITDA}},$$

$$\text{RCF to net debt} = \frac{\text{RCF}}{\text{Debt} - \text{Cash and marketable securities}}$$

FFO = net income from continuing operations + D&A + deferred income tax + other non-cash items; RCF = net cash from operating activities – dividends paid; FCF = net income – working capital increases – investment in fixed assets – net interest paid; Debt/capital = Debt/(Debt + Equity). Variants: EBITDA or EBITDAR in the numerator of coverage; interest may be reduced by interest earned on cash. These are non-IFRS measures with several possible definitions. Expected loss = POD × LGD = POD × (1 – RR) per unit of exposure.

Worked example (CFA curriculum)

Bowstream stress scenario (book ratios, Year 0 → Year 5). EBIT margin 25% → 5%; EBIT/interest 51.9× → 5.2×; Debt/EBITDA 0.36× → 1.21× (1,104/916); RCF/net debt 379% → 54% (Year 5: 377/(1,104 – 400)). Weaker profitability and coverage plus higher leverage point to wider spreads and a likely downgrade.

FFO/debt. Net income 180 + D&A 30 + deferred taxes 18 + other non-cash 28 = FFO 256; debt 1,280, so FFO/debt = **20%** (working capital, dividends and capex are not in FFO).

Priority of claims. Assets: cash 40, net current assets 100, fixed assets 60. Claims: bank loans 30 (secured by the cash), unsecured bonds 90, subordinated bonds 10, equity 70. A USD90 impairment of net current assets is absorbed by equity (70), then subordinated bonds (10, **zero recovery**), then 10 by the unsecured bonds, which recover **80 of 90**; the secured loan is untouched.

EL by seniority. POD 0.8%; recovery 58% / 46% / 23%: EL = 0.008 × 0.42 = **0.336%** (senior secured), **0.432%** (senior unsecured), **0.616%** (subordinated). [Note: book cites Exhibit 5; 58% and 46% match the automotive row of Exhibit 6, but its subordinated figure is 49.46%, not 23%]

What examiners like to test

- **Direction of leverage ratios:** a *higher* debt/EBITDA means more leverage, but a *higher* RCF/net debt or FFO/debt means *less*. Debt-funded buybacks, special dividends and acquisitions raise credit risk.
- **Issuer vs issue rating:** two senior unsecured bonds of one issuer have the same issuer and issue ratings whatever their maturity; subordinated debt of a B+ issuer is typically notched below (e.g., B–). Cross-default gives all issues the same POD; ratings differ through LGD.
- **Structural subordination:** operating-subsidiary debt is paid from subsidiary cash flows before anything is upstreamed, so holding-company bonds usually recover less.

Module 17 — Fixed-Income Securitization

Topic: Fixed Income · Exam relevance: **Low-yield · Concept-heavy**

The big picture

Securitization turns a pool of illiquid loans or receivables into tradable bonds. The originator sells the pool to a legally separate, **bankruptcy-remote** special purpose entity (SPE), which issues asset-backed securities (ABS) and pays investors from the pool's cash flows. Because the SPE is cut off from the originator, investors take the credit risk of the **borrowers in the pool**, not of the company that made the loans.

Learning outcomes in plain language

- LOS 1 Explain the benefits of securitization.** *Issuers:* move assets and lending risk off the balance sheet (lower leverage, less capital tied up), sell illiquid assets, earn fees, fund at lower cost and expand origination beyond the balance sheet. *Investors:* a direct conduit to borrowers; tailor interest rate and credit exposure to risk, return and maturity needs; access to diversified, imperfectly correlated assets they could not originate themselves; bond-like, tradable securities. *Economies and markets:* more liquid, tradable securities and price discovery, better system liquidity and less liquidity risk, and an alternative to traditional bond and equity funding. Risks remain: timing (contraction, extension) and credit risk of the pool.
- LOS 2 Describe securitization, its parties and their roles.** Originator makes loans, pools them and sells them to the SPE (true sale; assets derecognised); SPE issues ABS to investors; borrowers' payments flow through the SPE to investors. Three main parties: the **seller/depositor**, the **SPE** (called the issuer in the prospectus) and the **servicer**. Third parties: trustee, accountants, lawyers, underwriters, rating agencies, sometimes guarantors. Key documents: **purchase agreement** (seller's representations and warranties) and **prospectus** (structure, payment priority, credit enhancements).

Key facts

Products in order of increasing complexity

Covered bonds	Simplest; bank ring-fences a mortgage pool but it stays on its balance sheet; not a full securitization
Pass-through securities	True securitization; pool moved to an independent entity and payments passed through to investors
Structurally enhanced bonds	Cash flows redistributed across tranches on a preset schedule; subordination and credit enhancements (e.g., CMOs)
MBS vs non-mortgage ABS	RMBS, CMBS, CMO vs CDO, CLO, CBO, CDO-squared (US distinction)

Parties and roles

Seller / depositor	Originates the loans and sells them to the SPE (e.g., BRWA)
SPE / SPV (issuer)	Buys the pool, issues the ABS; legal separation protects investors if the seller fails
Servicer	Collects payments, contacts delinquent borrowers, repossesses and sells collateral
Trustee	"Disinterested" institution that safeguards the assets, holds funds due and reports cash flows to ABS holders

Why ABS can fund more cheaply than a secured corporate bond: the SPE is not affected by the seller's bankruptcy, and courts in most jurisdictions cannot change seniority. Transfers can still be challenged as fraudulent conveyances, and legal frameworks (e.g., trust law) differ across countries.

Worked example (CFA curriculum)

BRWA car loan securitization. BRWA's finance subsidiary holds EUR1,000 million of fixed-rate, fully amortizing car loans (45,000 loans, average balance EUR22,222). Steps: (1) BRWA sells cars financed by loans; (2) sells the loans to Car Loan Trust (CLT), an SPE; (3) receives EUR1,000 million cash; (4) CLT issues ABS; (5) investors pay cash; (6) borrowers' monthly payments go to CLT; (7) CLT pays ABS investors. BRWA's subsidiary stays on as servicer; capital tied up in loans is freed for the core business.

Ahbaling Trust. A below-investment-grade machine-tool maker with SGD500 million of bonds sells SGD400 million of receivables to Ahbaling Trust, which issues Class A (senior) SGD280 million and Classes B and C (subordinated) SGD60 million each. ABS buyers rely on the **customers'** payments, not on Ahbaling's health; if Ahbaling later issues new bonds, seniority does not apply between them and the ABS (different entities). The ABS pay a lower rate mainly because they are secured by the receivables.

What examiners like to test

- **Bankruptcy of the originator:** ABS investors are not affected (bankruptcy-remote SPE); their only credit risk is default by the underlying borrowers.
- **Match benefit to party:** tailoring risk and matching liability dates = investors; freeing capital, fee income, more lending = issuers; liquidity and price discovery = markets. ABS resemble bonds, not equity.
- **Order of steps:** set up the SPE and sell the assets to it *before* issuing ABS; servicing continues whoever does it.

Module 18 — Asset-Backed Security (ABS) Instrument and Market Features

Topic: Fixed Income · Exam relevance: **Medium-yield · Calculation + concept**

The big picture

Every ABS structure re-cuts one pool of cash flows into slices with different risks. **Covered bonds** keep the pool on the bank's balance sheet and give investors two lines of defence. True ABS move the pool off balance sheet and protect senior investors with **credit enhancement**: extra collateral, excess spread and, above all, subordination (the "waterfall"). CDOs apply the same idea to pools of bonds or loans, with a manager actively running the collateral.

Learning outcomes in plain language

- LOS 1 Covered bonds vs other ABS.** Senior debt of a financial institution backed by a ring-fenced **cover pool** (mortgages, public-sector assets) that stays on the balance sheet; **dual recourse** (cover pool first, then the issuer's unencumbered assets); usually one bond class; **dynamic** pool (non-performing or prepaid assets replaced); overcollateralization and LTV limits. Redemption regimes: **hard bullet** (missed payment = default and acceleration), **soft bullet** (default deferred to a new final maturity, usually up to a year later), **conditional pass-through** (converts to a pass-through). Result: lower credit risk and lower yield than similar ABS.
- LOS 2 Credit enhancement structures.** Internal: **overcollateralization** (collateral > bonds issued), **excess spread** (collateral coupon – securities coupon, absorbs losses or builds reserves) and **subordination/credit tranching** (junior classes absorb losses first). External (not covered in detail): bank or insurer guarantees, letters of credit, cash collateral accounts. Tranche ratings reflect pool quality and seniority, and can exceed the originator's rating.
- LOS 3 Non-mortgage ABS: cash flows and risks.** *Amortizing* collateral (auto loans): scheduled principal plus prepayments, exposed to prepayment and default risk from day one. *Non-amortizing* collateral (credit card receivables): during the **lockout/revolving** period principal repaid is reinvested and investors receive only finance charges and fees; then an amortization period; **rapid (early) amortization** triggers protect investors. *Solar ABS*: prime homeowner borrowers, often junior liens, green-bond appeal, pre-funding period.
- LOS 4 CDOs: cash flows and risks.** Backed by diversified debt: CBOs (bonds), CLOs (leveraged bank loans; the dominant type), structured finance CDOs, synthetic CDOs. A **collateral manager** buys and sells assets; the deal is a leveraged bet that collateral returns exceed funding costs. Senior/mezzanine tranches earn fixed returns; the unrated **equity** tranche takes residual cash flows and decides viability. Coverage tests (e.g., OC tests) divert cash to senior tranches when breached; extra risk: the manager fails to earn enough to pay senior and mezzanine holders.

Key formulas

$$\text{Overcollateralization ratio} = \frac{\text{Principal value of collateral}}{\text{Principal promised to debt tranches}}, \quad \text{Excess spread} = \text{Collateral coupon} - \text{Securities coupon}$$

Losses hit the most junior tranche first, then move up the capital structure; a tranche loses only after everything below it is wiped out.

Worked example (CFA curriculum)

Car Loan Trust waterfall (EUR1,000 million pool): A 825 (MRR + 0.50%), B 100 (MRR + 1.50%), C 50 (MRR + 2.50%), D 25 (variable). A collateral loss of EUR70 million wipes out D (25) and costs C 45, so C is repaid only 5; A and B lose nothing. A is hit only if losses exceed **EUR175 million**. With MRR = 4%, an investor with EUR3 million of B and EUR2 million of C gets interest of $3 \times 5.5\% = \text{EUR}165$ thousand and $2 \times 6.5\% = \text{EUR}130$ thousand. Defaults of EUR20 million are absorbed by D, so interest is unchanged; with defaults of EUR90 million, C is wiped out (0) and B loses 15 of 100, so B pays $165 \times 0.85 \approx \text{EUR}140$ thousand.

Other enhancements. Collateral EUR1,200 million behind EUR1,000 million of notes: after 10% defaults EUR1,080 million remains. Loans at 8% funding notes at 6%: excess spread 2%. CLO with USD840 million of loans behind USD700 million of debt: OC ratio **1.20**. Credit card ABS: issue 900 with 25 of overcollateralization and a 50 junior tranche: losses must exceed 75 (**8.33%**) before Tranche C is hit. [Note: the book labels this deal CHF900 million but states the tranche face values in EUR]

What examiners like to test

- **Waterfall arithmetic:** allocate losses bottom-up (e.g., SGD140 million loss on 280/60/60 leaves A = 260, B = 0, C = 0). Credit tranching protects *senior* classes, not the equity tranche.
- **Covered bond vs ABS:** on balance sheet, dual recourse, one class, dynamic pool; hard bullet defaults fastest. A CDO's collateral pool is not static; an MBS pool is.
- **Revolving period:** card ABS holders receive only finance charges and fees; principal is reinvested unless rapid amortization is triggered.

Module 19 — Mortgage-Backed Security (MBS) Instrument and Market Features

Topic: Fixed Income · Exam relevance: **Medium-yield** · Calculation + concept

The big picture

A mortgage borrower can usually repay early, so an MBS investor has effectively sold the borrower an option: when rates fall the principal comes back too soon (**contraction**), and when rates rise it comes back too late (**extension**). Pass-throughs simply hand this risk to investors; CMOs **time-tranche** it so each class gets the exposure it wants. Commercial MBS swap much of the prepayment risk for concentration and **balloon** risk.

Learning outcomes in plain language

- LOS 1 Define prepayment risk and time tranching.** Prepayment risk = principal repaid at a pace different from schedule. **Contraction risk** (rates fall, refinancing speeds up; reinvest at lower rates) and **extension risk** (rates rise, repayment slows). **Time tranching** creates classes with different expected maturities (e.g., sequential pay) to redistribute prepayment risk.
- LOS 2 Describe features of residential mortgages that are securitized.** First lien on the property; **LTV** (lower LTV = more borrower equity, less default, more lender protection) and **DTI**; prime vs subprime; agency RMBS (government agency or GSE guarantee) vs non-agency RMBS (credit enhancement instead); prepayment option and penalties (common in Europe, rare in the US); **recourse** vs **non-recourse** loans and strategic default on “underwater” (LTV > 100%) mortgages.
- LOS 3 Describe RMBS: pass-throughs and CMOs, cash flows and risks.** Pass-through: monthly principal, interest and prepayments passed through less fees, so the **pass-through rate** is below the WAC; use WAC, WAM and weighted average life. CMOs re-tranche pass-through cash flows: **sequential pay** (A protected from extension by B and C; B and C protected from contraction by A), **PAC/support**, Z (accrual), PO, IO, floating and residual tranches. CMOs redistribute, but cannot eliminate, prepayment risk.
- LOS 4 Describe CMBS characteristics and risks.** Few, large loans on income-producing property: **concentration risk**, so analyse each loan, property and owner. Call protection at the **structural** level (sequential tranches) and **loan level** (prepayment lockout, prepayment penalty points, **defeasance**), so CMBS trade more like corporate bonds. Balloon maturities create **balloon risk**, a form of extension risk. Key credit metrics: LTV and **DSC ratio**; commercial mortgages are non-recourse.

Key formulas

$$LTV = \frac{\text{Loan amount}}{\text{Property value}}, \quad DTI = \frac{\text{Monthly debt payment}}{\text{Monthly pre-tax gross income}}, \quad DSC = \frac{\text{Net operating income}}{\text{Debt service}}$$

$$WAC = \sum_j i_j \left(\frac{CB_j}{\sum CB} \right), \quad WAM = \sum_j MM_j \left(\frac{CB_j}{\sum CB} \right)$$

where i_j = mortgage rate; CB_j = **current** (not beginning) balance; MM_j = months to maturity; NOI = (rental income – cash operating expenses) – replacement reserves; debt service = annual interest plus principal. DSC > 1.0× means property cash flow covers debt service. Pass-through rate = WAC – servicing and other fees (e.g., 4.55% – 0.55% = 4.00%).

Worked example (CFA curriculum)

LTV and DTI. House EUR800,000, loan EUR600,000 at 4.50% over 240 months, gross income EUR141,000. LTV = 600,000/800,000 = **75%**; monthly payment EUR3,795.90; DTI = 3,795.90/(141,000/12) = **32.31%**.

WAC and WAM. Five mortgages with current balances 238, 380, 87, 132, 312 (EUR000; total 1,149), rates 2.50/3.30/2.80/4.00/3.70%, months to maturity 180/480/240/120/312: WAC = **3.29%**, WAM = **313** months.

Sequential-pay CMO on a USD100 million, 4% pass-through: A USD50m at 3.75% (years 0–5), B USD30m at 3.95% (6–15), C USD20m at 4.05% (16–30). Extension-averse investors buy A; long-term, return-seeking investors buy C.

DSC. Office building (FMV GBP27m, GBP21m 5-year loan at 4.0%): NOI GBP7,590,000; debt service GBP4,717,169; DSC = **1.61×**.

What examiners like to test

- **Contraction vs extension:** falling rates → faster prepayment → contraction (MBS maturity shortens); rising rates → extension. PO values rise and IO cash flows fall when prepayments speed up.
- **Weighting trap:** WAC and WAM use *current* balances; using beginning balances or a simple average gives the wrong answer (e.g., 4.50% or 4.00% instead of 4.73%).
- **CMBS vs RMBS:** CMBS have less prepayment risk but more concentration (credit) risk; RMBS pools are large and diversified.