

CFA® Level I

One-Page Learning Module Summaries

Derivatives

10 learning modules · one page each · built from CFA Program Level I curriculum, Volume 7

How each page works

■ Module banner	Exact module title, exam relevance (High/Medium/Low-yield) and whether it is calculation- or concept-heavy.
■ The big picture	Two or three sentences of intuition: why the module exists.
■ Learning outcomes	Every LOS of the module, in book order, rewritten as what you must know or do.
■ Key formulas / key facts	Formulas in curriculum notation with every variable defined; fact tables for concept modules.
■ Worked example	One numerical example (or a short Ethics case). Tagged [CFA curriculum] when taken from the curriculum, [Illustrative] when made up by RBei.
■ What examiners like to test	Traps, key distinctions and typical question angles.

Notes. (1) The curriculum lists LOS without letters; “LOS 1, 2, 3” follows the order in the book. (2) Exam relevance is RBei’s judgement from the number of LOS, the calculation LOS and the weight of worked examples and practice problems in each module; the curriculum does not rate modules. (3) [India context, not curriculum] marks Indian examples added for context; they are not curriculum. (4) [Note: ...] marks places where the curriculum is inconsistent or offers two framings.

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Module 1 — Derivative Instrument and Derivative Market Features

Topic: Derivatives · Exam relevance: Medium-yield · Concept-heavy

The big picture

A derivative is a contract whose value comes from something else — a share, an index, an interest rate, a currency, a commodity or a borrower's credit. It does not pass the underlying's return straight through; it *transforms* it, letting users create, modify or hedge exposure without trading the asset today. The market question is *where* the contract lives: a customised bilateral deal over the counter (OTC), or a standardised, margined exchange contract — and, since the 2008 crisis, most OTC trades between dealers are pushed through a central counterparty.

Learning outcomes in plain language

- LOS 1 Define a derivative and describe its basic features.** Underlying; counterparties (buyer $\approx$ long exposure, seller $\approx$ short exposure); maturity and settlement; contract size (notional); pre-agreed price. *Firm commitment* (forward, futures, swap) vs. *contingent claim* (option); stand-alone vs. *embedded* (callable, puttable, convertible bond). Common underlyings: equities, fixed income and interest rates (e.g. MRR such as SOFR), currencies, commodities (soft and hard), credit, and “other” (weather, crypto, longevity); hedging = offsetting an existing or anticipated exposure.
- LOS 2 Describe derivative markets and contrast OTC with exchange-traded (ETD) markets.** OTC = customised contracts between end users and dealers (market makers who offset risk with each other); ETD = standardised terms set by the exchange, clearing, settlement, collateral (margin) and exchange guarantee, transparency; central clearing of OTC trades through a CCP by *novation*.

Key facts

	Over-the-counter (OTC)	Exchange-traded (ETD)
Terms	Customised (size, underlying, maturity, credit terms)	Standardised by the exchange
Counterparty risk	Usually more; privately negotiated credit terms	Collateral on deposit; exchange/clearinghouse guarantee
Transparency	Less (more privacy/confidentiality)	Full disclosure to exchange and regulators
Liquidity and cost	May be less liquid	More liquid, lower trading and transaction costs
Market makers	Yes (dealers)	Yes (exchange members)

Why use derivatives: sell short; diversify; offset exposure from a commercial transaction; large exposure for a relatively *small* cash outlay; lower transaction costs and often more liquid than the spot market.

Clearing = verifying execution, exchanging payments, recording participants. **Settlement** = paying final amounts and/or delivering securities or commodities.

Central clearing (post-2008 mandate): (1) trade executed on a swap execution facility (SEF) → (2) details sent to the CCP → (3) CCP *novates* the trade and faces both dealers. Benefit: bilateral counterparty risk removed, ETD-like margining; cost: risk *centralised and concentrated* in CCPs (systemic risk).

Worked example (CFA curriculum)

(a) Forward on Airbus shares. AMY Investments agrees today to deliver 1,000 Airbus (AIR) shares in six months at a fixed EUR 30 per share (firm commitment, stand-alone, counterparty a financial intermediary). If $S_T = \text{EUR } 25$, AMY receives $1,000 \times 30 = \text{EUR } 30,000$ for shares worth $1,000 \times 25 = \text{EUR } 25,000$ — or the two sides simply cash-settle the **EUR 5,000** difference. AMY has transferred its AIR price risk but now bears the intermediary's *counterparty credit risk*.

(b) Which market? Montau AG (Germany) will receive KRW 650,000,000 from a Korean buyer in 75 days. The best hedge is a *firm commitment* to sell KRW at a fixed EUR price, traded **OTC**, because only a customised contract matches the exact amount and 75-day date. Hightest Capital's six-month call on a health-care sector index (standard size, exercise price and maturity) would most likely trade on an **exchange**; Esterr Inc.'s CAD 250m pay-fixed swap, tailored to its loan dates, trades OTC and is then centrally cleared between dealers.

What examiners like to test

- **Classify the contract:** forwards, futures and swaps are firm commitments; options are contingent claims; a callable bond contains an *embedded* contingent claim; buying an index ETF is a *cash/spot* trade, not a derivative.
- **Wording trap:** “large exposure for a relatively *small* cash outlay” is a benefit of derivatives; “large exposure for a *similar* cash outlay” is not.
- **Credit risk under central clearing:** the CCP takes on each side's counterparty risk, so the main worry becomes *systemic* (can the CCP pay?). The curriculum's answer: an end user still poses credit risk *to* its counterparty.

Module 2 — Forward Commitment and Contingent Claim Features and Instruments

Topic: Derivatives · Exam relevance: High-yield · Calculation + concept

The big picture

Forwards, futures and swaps are *firm commitments*: both sides must perform, nothing is paid up front, and the payoff is a straight line in the underlying price. Options are *contingent claims*: the buyer pays a premium for the right, not the obligation, so the payoff is bent — zero on one side, rising on the other. Most questions come down to writing the payoff correctly, subtracting the premium to get profit, and knowing who bears whose credit risk.

Learning outcomes in plain language

- LOS 1 Define and compare forwards, futures, swaps, options and credit derivatives.** Forward = OTC, customised, usually one exchange at T ; futures = standardised ETD with daily mark to market (MTM), initial and maintenance margin, clearinghouse guarantee, price limits; swap = series of exchanges (fixed vs. floating MRR, netted, notional not exchanged); call = right to buy, put = right to sell, European (exercise at maturity) vs. American (any time); CDS = protection buyer pays a fixed spread, seller pays about $\text{LGD} \times \text{notional}$ on a credit event.
- LOS 2 Determine the value at expiration and the profit** of a long or short call or put, including breakeven, maximum gain and maximum loss.
- LOS 3 Contrast forward commitments with contingent claims:** linear/symmetric vs. non-linear/asymmetric payoffs; both sides vs. only the option buyer bearing counterparty risk; long forward vs. long call vs. short put when $X = F_0(T)$.

Key formulas

Notation: $F_0(T)$ = forward price set at $t = 0$ for exchange at T ; $f_t(T)$ = futures price on day t ; S_T = spot price at T ; X = exercise price; c_0, p_0 = call and put premiums paid at $t = 0$; Π = profit (time value of money ignored, as in the curriculum).

Long forward payoff = $S_T - F_0(T)$ (short: $F_0(T) - S_T$) Daily futures MTM (long) = $[f_t(T) - f_{t-1}(T)] \times \text{size}$

$c_T = \max(0, S_T - X)$, $\Pi = \max(0, S_T - X) - c_0$ $p_T = \max(0, X - S_T)$, $\Pi = \max(0, X - S_T) - p_0$

Short positions: $-c_T + c_0$ and $-p_T + p_0$. **Breakeven:** long call $X + c_0$; long put $X - p_0$. **Max loss:** long option = premium; short call unlimited; short put $X - p_0$ (at $S_T = 0$). **Moneyness (LM2 wording):** call in the money if $S_t > X$, intrinsic value $S_t - X$; ATM and OTM options are pure time value.

Swap (fixed-rate payer) net receipt = $(\text{MRR} - \text{fixed rate}) \times \text{notional} \times \text{period}$. **CDS:** payment on credit event $\approx \text{LGD} \times \text{notional}$; MTM change $\approx \Delta \text{CDS spread} \times \text{notional} \times \text{EffDur}_{\text{CDS}}$. **Margin call:** if the balance falls below maintenance margin, deposit variation margin back up to the *initial* margin.

Worked example (CFA curriculum)

(a) Call payoff and profit. Hightest buys a six-month SIXV index call, $X = \$1,240$, $c_0 = \$24.85$ per unit (100 units: \$2,485). At $S_T = \$1,280$: payoff $\max(0, 1,280 - 1,240) = \$40$, profit $40 - 24.85 = \$15.15$; at \$1,260 profit is $-\$4.85$; at or below \$1,240 it is $-\$24.85$. Breakeven = $1,240 + 24.85 = \$1,264.85$.

(b) NIFTY 50 call. $X = \text{INR } 16,000$, premium $\text{INR } 1,500$, index at $\text{INR } 16,500$ at maturity: payoff **INR 500**, profit $500 - 1,500 = -\text{INR } 1,000$.

(c) Futures margin call. Procam is long one 100-oz gold contract; initial margin \$4,950, maintenance \$4,500, balance \$5,450 after day one. The price falls from \$1,797.13 to \$1,786.25: loss = $100 \times 10.88 = \$1,088$; balance $5,450 - 1,088 = \$4,362 < \$4,500$, so the margin call is $4,950 - 4,362 = \$588$ (back to initial, not to maintenance). Forward and futures both lose \$1,163 in total at $S_T = \$1,780.50$; only the *timing* differs.

(d) Swap netting. Fyleton receives 2.25% fixed and pays six-month MRR (1.95%) semiannually on GBP 200m: receives GBP 2,250,000, pays GBP 1,950,000, so it **receives GBP 300,000 net**.

What examiners like to test

- **Payoff vs. profit:** a call with $X = 45$, premium 6, $S_T = 50$ has payoff 5 but profit -1 (seller $+1$). A put buyer profits only if $S_T < X - p_0$.
- **Same-direction positions:** long forward, long call and short put gain when the price rises; short forward, short call and long put gain when it falls.
- **Credit risk:** an option *seller* has no credit exposure once the premium is paid; in a forward both sides are exposed. The CDS protection *buyer* is short credit risk; the seller is like a long bondholder.

Module 3 — Derivative Benefits, Risks, and Issuer and Investor Uses

Topic: Derivatives · Exam relevance: Medium-yield · Concept-heavy

The big picture

Derivatives let people move risk across time and between parties without trading the underlying: an exporter fixes tomorrow's exchange rate today, a fund changes duration without selling bonds. The same features that make them cheap and efficient — little cash up front, easy shorting — create **implicit leverage**, basis, liquidity and counterparty risk. Issuers mostly *hedge* (and want hedge accounting); investors hedge, replicate or add exposures and mark everything to market daily.

Learning outcomes in plain language

- LOS 1 Describe benefits and risks of derivatives.** Benefits: risk allocation, transfer and management (bridge the timing gap between an economic decision and the cash trade; create exposures unavailable in cash markets, e.g. covered call); information/price discovery (futures prices, implied volatility — not an unbiased forecast); operational advantages (lower transaction costs, more liquidity, low upfront cash, easy shorting); market efficiency. Risks: implicit leverage/speculative use, lack of transparency (e.g. structured notes), basis risk, liquidity risk, counterparty credit risk, destabilisation and systemic risk.
- LOS 2 Compare derivative use by issuers and investors.** Issuers hedge exposures incidental to operations and financing, prefer customised OTC contracts that qualify for **hedge accounting** (cash flow, fair value, net investment). Investors replicate cash strategies, hedge fund value, or modify/add exposures; they care less about hedge accounting (positions are in daily NAV) and use liquid ETDs more.

Key facts

Risk	Curriculum definition	
Basis risk	Divergence between the expected <i>value</i> of a derivative and the underlying/hedged item (e.g. different MRR, CDS vs. bond spread)	
Liquidity risk	Divergence in the <i>cash flow timing</i> of the derivative vs. the hedged item (e.g. futures margin calls)	
Counterparty credit risk	Counterparty fails to perform; forward: both sides exposed; bought option: only the buyer exposed	
Systemic risk	Excessive risk taking and leverage contribute to market stress (2008 crisis)	
Hedge designation	What it offsets	Examples
Cash flow	Variable cash flows of a floating-rate asset/liability or forecast transaction	Pay-fixed swap on floating-rate debt; FX forward on forecast sales
Fair value	Changes in fair value of an asset or liability	Receive-fixed swap on fixed-rate debt; commodity futures on inventory
Net investment	FX risk of the equity of a foreign operation	Currency swap or forward
Under hedge accounting, derivative MTM changes are held in OCI and released when the hedged item hits earnings; terms (dates, notional) must closely match, which is why issuers favour OTC.		

Worked example (CFA curriculum)

Implicit leverage (Procam gold). Spot purchase: 100 oz at \$1,770 = \$177,000 cash. Futures: $f_0(T) = \$1,792.13$, initial margin \$4,950. At $S_T = \$1,780.50$ both positions are worth \$178,050.
 Spot: $(178,050 - 177,000)/177,000 = +0.593\%$. Futures: $(178,050 - 179,213)/4,950 = -23.5\%$.
 Financing both at 5% for three months: interest on margin = $4,950 \times 0.05/4 = \$61.88$; on spot = $177,000 \times 0.05/4 = \$2,212.50$.
 Return on financing cost: spot +47.5%, futures -1,879%; exposure of \$177,000 for \$61.88 is a leverage ratio of about **2,860**.
Issuer hedge (Montau AG). Selling KRW 650m forward at 1,350 KRW/EUR locks in EUR 481,481 against a cost of EUR 430,000: margin about **11%** at every future spot rate, versus -1% (at 1,525) to 19% (at 1,225) unhedged — a cash flow hedge.

What examiners like to test

- **Basis vs. liquidity risk:** basis = *value* mismatch; liquidity = *timing* mismatch of cash flows. Swapped definitions are a favourite distractor.
- **Hedge type:** floating-to-fixed swap = cash flow hedge; fixed-to-floating swap = fair value hedge; hedge of a foreign subsidiary's equity = net investment hedge.
- **Structured notes:** an "80% principal protected" note linked to an index embeds a *long call*; the investor bears the issuer's credit risk on 100% of face value, with more cost, less liquidity and transparency than the exchange-traded option.

Module 4 — Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

Topic: Derivatives · Exam relevance: **High-yield** · Calculation + concept

The big picture

A forward price is not a forecast. It is whatever price stops anyone locking in a riskless profit by buying the asset with borrowed money and selling it forward (or the reverse). So the forward price is the spot price carried forward at the risk-free rate, adjusted for anything that costs you (storage, insurance) or pays you (dividends, coupons, foreign interest, convenience yield) while you hold the asset.

Learning outcomes in plain language

- LOS 1 Explain how arbitrage and replication price derivatives.** Law of one price: assets with identical future cash flows must have the same price, and an asset with a known future price must trade at its PV at the risk-free rate. Replication: long forward = borrow S_0 at r and buy the asset; short forward = short the asset and lend; long asset + short forward earns r . Know the arbitrage steps when $F_0(T)$ is above or below $S_0(1+r)^T$.
- LOS 2 Explain spot vs. future price and the cost of carry.** The gap between spot and forward is the net cost of carry: opportunity cost r (every asset), other costs C/c (commodities), benefits I/i (dividends, coupons, dividend yield, foreign rate, convenience yield); FX forwards depend on $r_f - r_d$.

Key formulas

Compounding convention (curriculum): discrete $(1+r)^T$ for individual assets; **continuous** e^{rT} for index underlyings (equity, fixed-income, commodity, credit indexes) and FX. The risk-free rate for most participants is the repo rate.

$$\text{Discrete: } F_0(T) = S_0(1+r)^T, \quad F_0(T) = [S_0 - PV_0(I) + PV_0(C)](1+r)^T$$

$$\text{Continuous: } F_0(T) = S_0e^{rT}, \quad F_0(T) = S_0e^{(r+c-i)T}, \quad F_{0,f/d}(T) = S_{0,f/d}e^{(r_f-r_d)T}$$

where I, C = income and costs as known amounts (PV at $t = 0$); i, c = income and cost as rates of return (e.g. index dividend yield); $S_{0,f/d}$ = units of price (foreign, f) currency per one unit of base (domestic, d) currency; r_f, r_d = their risk-free rates.

Direction: costs (incl. $r > 0$) > benefits $\Rightarrow F_0(T) > S_0$; benefits > costs $\Rightarrow F_0(T) < S_0$; $r < 0 \Rightarrow F_0(T) < S_0$. FX: $r_f > r_d \Rightarrow F > S$, price currency at a forward **discount**, base at a premium. A forward price well *below* the no-arbitrage level may signal a convenience yield.

Arbitrage: if $F_0(T) > S_0(1+r)^T$: borrow, buy spot, sell forward, profit $F_0(T) - S_0(1+r)^T$. If $F_0(T) < S_0(1+r)^T$: short spot, lend, buy forward.

Worked example (CFA curriculum)

Arbitrage (Procam gold). $S_0 = \$1,770$, $r = 2\%$, no storage cost, forward sale at \$1,792.13 in three months. Borrow \$177,000, buy 100 oz; repay $177,000(1.02)^{0.25} = \$177,878.44$; deliver for \$179,213: riskless profit **\$1,334.56** (\$13.35/oz). No arbitrage requires $S_0 = 1,792.13(1.02)^{-0.25} = \$1,783.28$; adding \$2/oz storage paid at T : $F_0 = (1,783.28 + 1.99)(1.02)^{0.25} = \mathbf{\$1,794.13}$.

Dividend (discrete). Unilever $S_0 = \text{EUR } 50$, $r = 5\%$, six months: no dividend $F_0 = 50(1.05)^{0.5} = \text{EUR } 51.23$. With EUR 0.30 at 3 and 6 months: $PV_0(I) = 0.2964 + 0.2928 = 0.5892$; $F_0 = (50 - 0.5892)(1.05)^{0.5} = \mathbf{\text{EUR } 50.6310}$.

Index (continuous). NIFTY 50 at INR 15,200, dividend yield 2.2%, $r = 4\%$, three months: $F_0 = 15,200e^{(0.04-0.022)(0.25)} = \mathbf{\text{INR } 15,268.55}$.

FX (continuous). AUD/USD spot 1.3335, $r_{AUD} = 0.05\%$, $r_{USD} = 0.20\%$, six months: $F = 1.3335e^{(0.0005-0.002)(0.5)} = \mathbf{1.3325}$ — fewer AUD per USD, so the AUD trades at a forward premium.

What examiners like to test

- **Compounding choice:** a single stock or gold uses $(1+r)^T$; an index or an exchange rate uses $e^{(\cdot)T}$. Mixing them gives slightly wrong answers that appear as distractors.
- **FX direction:** put the *price* currency's rate first ($r_f - r_d$). Self-check: the higher-rate currency is at a forward discount.
- **Arbitrage size:** spot GBP 50, 6-month forward 52.50, $r = 4\%$: $52.50 - 50(1.04)^{0.5} = \mathbf{\text{GBP } 1.51}$ (the distractor GBP 0.99 is the no-arbitrage forward minus spot).

Module 5 — Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Topic: Derivatives · Exam relevance: High-yield · Calculation-heavy

The big picture

Keep **price** and **value** apart. The forward *price* $F_0(T)$ is fixed at inception and never changes; the contract's *value* starts at zero, moves as the spot price, rates and time change, and ends as the settlement amount. For interest rates the “asset” has a term structure, so we bootstrap zero rates, link them with implied forward rates, and use those rates to price FRAs.

Learning outcomes in plain language

- LOS 1** Explain the value and price of a forward at initiation, during its life and at expiration. $V_0(T) = 0$; $V_t(T) =$ spot (adjusted for remaining carry) minus PV of $F_0(T)$ for the long, reverse for the short; $V_T(T) = S_T - F_0(T)$. Know the effect of a spot change, the passage of time and a change in r (or in $r_f - r_d$ for FX).
- LOS 2** Explain how forward rates are set for interest rate forwards and their uses. Bootstrap zero rates from coupon bonds; discount factors; implied forward rate (IFR) as the breakeven reinvestment rate; “AyBy” notation; periodicity conversion; FRA = OTC contract fixing a rate for a future period, priced at the IFR, cash settled at the start of the period (discounted at MRR).

Key formulas

Equity/commodity forwards (discrete, as the book writes them):

$$V_0(T) = 0, \quad V_t(T) = [S_t - PV_t(I) + PV_t(C)] - F_0(T)(1+r)^{-(T-t)}, \quad V_T(T) = S_T - F_0(T)$$

FX forward (continuous): $F_{0,f/d}(T) = S_{0,f/d} e^{(r_f - r_d)T}$, $V_t(T) = S_{t,f/d} - F_{0,f/d}(T) e^{-(r_f - r_d)(T-t)}$ (long base currency).

Short position = negative of each. A higher r lowers PV of $F_0(T)$: good for the long, bad for the short.

$$DF_i = \frac{1}{(1+z_i)^i}, \quad (1+z_A)^A(1+IFR_{A,B-A})^{B-A} = (1+z_B)^B, \quad \left(1 + \frac{APR_m}{m}\right)^m = \left(1 + \frac{APR_n}{n}\right)^n$$

FRA net payment (fixed payer, paid at B) = $(MRR_{B-A} - IFR_{A,B-A}) \times \text{Notional} \times \text{Period}$

$$\text{Cash settlement at } A = \frac{\text{Net payment}}{1 + MRR_{B-A} \times \text{Period}}$$

z_i = zero (spot) rate for period i ; $IFR_{A,B-A}$ = rate starting at A with tenor $B-A$ (“2y3y” = 3-year rate starting in 2 years). FRA long = fixed-rate payer, gains if MRR rises. FRAs are used almost exclusively by financial intermediaries.

Worked example (CFA curriculum)

Forward MTM (Biomian, VFO short). $S_0 = \text{INR } 295$, $F_0(T) = \text{INR } 300.84$, six months, implied $r = (300.84/295)^2 - 1 = 4\%$. Spot jumps instantly to 325: $V_0 = 295 - 325 = -\text{INR } 30$ for VFO. After three months with $S_t = 285$: $V_t = 300.84(1.04)^{-0.25} - 285 = \text{INR } 12.90$ gain; if r doubles to 8%: $300.84(1.08)^{-0.25} - 285 = \text{INR } 10.11$ (gain falls by 2.79). At T with $S_T = 287$: VFO +13.84, buyer -13.84.

Zero rates, IFR and FRA. Bonds: 1y 1.50% at 99.125, 2y 2.50% at 98.275, 3y 3.25% at 98.000 $\Rightarrow z_1 = 2.3960\%$, $z_2 = 3.4197\%$, $z_3 = 4.0005\%$; $IFR_{1,1} = 1.034197^2/1.02396 - 1 = 4.4536\%$, $IFR_{2,1} = 5.1719\%$. 3m MRR 1.25%, 6m MRR 1.75% (convert: $APR_4 = 1.74619\%$) $\Rightarrow IFR_{3m,3m} = 2.24299\%$. Yangzi Bank pays this fixed on CNY 100m; MRR sets at 2.15%: $(2.15\% - 2.24299\%) \times 100m \times 90/360 = -\text{CNY } 23,247.50$, settled at 3 months as $-23,247.50/(1 + 0.0215/4) = -\text{CNY } 23,123.21$ (Yangzi pays).

What examiners like to test

- **Sign by side:** long value = $S_t - PV(F_0)$; short = $PV(F_0) - S_t$. The forward *price* does not change during the contract — “the forward rate is adjusted” is always wrong.
- **Zero rate $\neq$ YTM:** bootstrap before computing IFRs (2y 3% coupon bond at 99 with $z_1 = 2.50\%$: $z_2 = 3.5422\%$, not the 3.5266% IRR that assumes a flat curve).
- **FRA settlement:** compute the end-of-period amount, then discount at the *realised* MRR to the start of the period.

Module 6 — Pricing and Valuation of Futures Contracts

Topic: Derivatives · Exam relevance: Medium-yield · Calculation + concept

The big picture

A futures contract is priced exactly like a forward at inception, but it is **marked to market every day**: gains and losses are paid through the margin account and the contract's value is reset to zero at each settlement price. The total gain or loss ends up about the same as on a forward; what differs is *when* cash moves — and that timing, together with how futures prices move with interest rates, is why futures and forward prices can differ.

Learning outcomes in plain language

- LOS 1 Compare the value and price of forward and futures contracts.** Both have zero value and the same no-arbitrage price at inception. Forward: price fixed, MTM $V_t(T) = S_t - PV(F_0(T))$ unrealised until T (counterparty risk builds up). Futures: price $f_t(T)$ changes daily, daily gain/loss settled through margin and value reset to zero; cumulative MTM $\approx$ forward. Interest rate futures quoted as 100–yield; BPV; short futures (gains if MRR rises) $\approx$ long (pay-fixed) FRA.
- LOS 2 Explain why forward and futures prices differ.** Identical if rates are constant or futures prices and rates are uncorrelated. Positive correlation $\Rightarrow$ long futures more attractive $\Rightarrow f_0(T) > F_0(T)$; negative $\Rightarrow F_0(T) > f_0(T)$; gap grows with rate volatility. Interest rate futures vs. FRAs: **convexity bias** (FRA discounting). Central clearing and dealer margining of OTC trades narrow the difference.

Key formulas

Futures price at inception (same convention as forwards — discrete for single assets, continuous for index/FX underlyings):

$$V_0(T) = 0, \quad f_0(T) = S_0(1+r)^T, \quad f_0(T) = [S_0 - PV_0(I) + PV_0(C)](1+r)^T, \quad f_0(T) = S_0e^{rT}$$

Daily settlement (long): gain = $[f_t(T) - f_{t-1}(T)] \times \text{contract size}$; value then resets to 0 at $f_t(T)$. Forward MTM: $V_t(T) = S_t - F_0(T)(1+r)^{-(T-t)}$ (not paid until T).

$$f_{A,B-A} = 100 - (100 \times MRR_{A,B-A}), \quad \text{Futures contract BPV} = \text{Notional} \times 0.01\% \times \text{Period}$$

$f_{A,B-A}$ = futures price on the MRR for $B - A$ periods starting at A . Long futures (lender) gains as MRR falls; short futures (borrower) gains as MRR rises. A price above 100 implies a negative expected MRR.

Condition	Preferred long position	Price relationship
Rates constant, or uncorrelated with futures prices	Indifferent	$f_0(T) = F_0(T)$
Futures prices positively correlated with rates	Futures (gains reinvested at higher rates, losses financed at lower rates)	$f_0(T) > F_0(T)$
Negatively correlated	Forward	$F_0(T) > f_0(T)$

Worked example (CFA curriculum)

Pricing. Gold $S_0 = \$1,770$, $r = 2\%$, 91 days: $f_0(T) = 1,770(1.02)^{91/365} = \$1,778.76$ (\$177,876 per 100 oz), identical to the forward price.

Forward vs. futures MTM. Day 1: futures fall \$5 to \$1,773.76 (no-arbitrage spot $1,773.76(1.02)^{-90/365} = \$1,765.12$). Futures: Procum loses \$500, paid in cash; balance $4,950 - 500 = \$4,450 < \$4,500$ maintenance, so it deposits \$500; contract value resets to 0. Forward: $V_t = 1,765.12 - 1,778.76(1.02)^{-90/365} = -\$4.98/\text{oz}$, a \$498 unrealised loss, nothing paid. [Note: the book's text prints +\$4.98; its own table shows -\$498]

Interest rate futures vs. FRA. Notional \$1,000,000, 3m MRR, BPV = $1,000,000 \times 0.01\% \times \frac{1}{4} = \$25$. If MRR moves from 2.21% to 2.01% / 2.41%, long futures settle +\$500 / -\$500, while the short FRA settles +\$497.50 / -\$497.01 (discounted at the realised MRR) — the non-linear FRA payoff is the **convexity bias**. Baywhite (\$50m, 1m MRR): BPV = \$416.67; selling at 98.75 and settling at 97.75 gains $416.67 \times 100 = \$41,667$.

What examiners like to test

- **Hedging a rise in MRR:** short interest rate futures or long (pay-fixed) FRA; a receive-fixed FRA $\approx$ long futures.
- **Realised vs. cumulative:** before maturity only futures gains are *realised*; at maturity cumulative results are only *approximately* equal (time value of money).
- **Margin call amount:** restore the balance to the *initial* margin, not to maintenance.

Module 7 — Pricing and Valuation of Interest Rates and Other Swaps

Topic: Derivatives · Exam relevance: Medium-yield · Calculation + concept

The big picture

An interest rate swap is a bundle of FRAs with one twist: instead of a different forward rate for each period, every period uses the **same fixed swap rate**. That rate is set so the swap is worth zero at the start — the PV of the fixed leg equals the PV of the expected floating leg. After that, the swap's value drifts with time and with the forward curve, and a receive-fixed swap behaves like a long fixed-rate bond.

Learning outcomes in plain language

- LOS 1 Describe how swaps are similar to but different from a series of forwards.** Similar: firm commitments, symmetric payoffs, zero value and no cash at inception, net settlement on a notional, counterparty credit risk. Different: a series of FRAs has a different fixed rate (the IFR) each period and each FRA settles at the *start* of its period; a standard swap has one constant fixed rate and settles at the *end* of each period. Issuers and investors prefer swaps (match periodic cash flows, one contract, liquidity); FRAs are mainly used by intermediaries.
- LOS 2 Contrast the value and price of swaps.** Price = par swap rate s_N (fixed at inception, an “IRR” of the implied forward rates). Value = 0 at inception; later = current settlement value + PV of remaining settlements. Rising forward rates $\Rightarrow$ MTM gain for the fixed-rate payer; with an upward-sloping curve, passage of time alone gives the payer a gain after early net payments. Pay-fixed $\approx$ long FRN + short fixed-rate bond (lower duration); receive-fixed $\approx$ long fixed-rate bond (higher duration).

Key formulas

$$\sum_{i=1}^N \frac{IFR_{i-1,1}}{(1+z_i)^i} = \sum_{i=1}^N \frac{s_N}{(1+z_i)^i} \iff \sum PV(\text{floating payments}) = \sum PV(\text{fixed payments})$$

Periodic settlement value (fixed-rate payer) = $(MRR - s_N) \times \text{Notional amount} \times \text{Period}$

z_i = zero rate for period i (discrete compounding, as in LM5); $IFR_{i-1,1}$ = implied one-period forward rate starting at $i-1$ ($IFR_{0,1} = z_1$); s_N = par swap rate for N periods. The curriculum assumes MRR sets at the start of each period and the net amount is paid at the end.

Position (bond equivalent)	Rates rise	Rates fall
Receive fixed, pay floating (long fixed bond, short FRN)	MTM loss	MTM gain
Pay fixed, receive floating (short fixed bond, long FRN)	MTM gain	MTM loss

Worked example (CFA curriculum)

Par swap rate. Zero rates 2.3960%, 3.4197%, 4.0005% give implied forwards $IFR_{0,1} = 2.396\%$, $IFR_{1,1} = 4.4536\%$, $IFR_{2,1} = 5.1719\%$. Discount factors 0.976601, 0.934961, 0.888982 (sum 2.800545). PV of floating = $\frac{0.02396}{1.02396} + \frac{0.044536}{1.034197^2} + \frac{0.051719}{1.040005^3} = 0.111017$, so $s_3 = 0.111017/2.800545 = \mathbf{3.9641\%}$. The equivalent FRAs would use 2.396%, 4.4536%, 5.1719%; the receive-fixed side of the swap expects +1.5681% in year 1 and pays net later.

Issuer use (Esterr, pay fixed). CAD 250m loan at MRR + 150 bp, swap pay 2.05% fixed, receive 3m MRR. If MRR = 3.75%: receives net 1.70%, loan costs 5.25% $\Rightarrow$ 3.55%. If MRR = 1.25%: pays net 0.80%, loan costs 2.75% $\Rightarrow$ 3.55%. Quarterly cost $\approx 3.55\% \times 250m \times 0.25 = \mathbf{CAD\ 2,218,750}$ either way. First settlements with IFRs 0.50% and 0.74%: $(0.50\% - 2.05\%) \times 250m \times 0.25 = -\mathbf{CAD\ 968,750}$; then $-\mathbf{CAD\ 818,750}$.

Investor use (Fyleton, receive 2.38% vs. 0.71% MRR, GBP 200m, semiannual): first settlement $(2.38\% - 0.71\%) \times 200m \times 0.5 = \mathbf{GBP\ 1,670,000}$ received.

What examiners like to test

- **Period factor:** 10-year pay-fixed EUR 100m at 1.12%, 6m MRR 0.25%: $(0.25\% - 1.12\%) \times 100m \times 0.5 = -\mathbf{EUR\ 435,000}$; forgetting $\times 0.5$ gives $\pm 870,000$.
- **Who gains when:** fixed-rate payer gains from higher expected forward rates; to gain from falling rates, *receive* fixed (like buying bonds, adds duration).
- **The swap rate, not MRR,** is the IRR on the implied forward rates; neither side knows its net cash flows at inception.

Module 8 — Pricing and Valuation of Options

Topic: Derivatives · Exam relevance: High-yield · Calculation + concept

The big picture

An option's price has two parts: what it would be worth if you could exercise now (**exercise value**) and the extra you pay for the chance that things improve before expiry (**time value**). Because the payoff is one-sided, options cannot be priced by a single fixed replication like forwards; instead we get no-arbitrage *bounds*, and a replication that must be adjusted as the chance of exercise changes. Six factors drive the value, and knowing their direction for calls and puts is core exam material.

Learning outcomes in plain language

- LOS 1 Explain exercise value, moneyness and time value.** Exercise value = value if exercisable now, using PV of X ; moneyness compares S_t with X (ITM, ATM, OTM; deeper ITM $\Rightarrow$ price moves nearly one-for-one with the underlying); time value = option price – exercise value, always positive, decays to zero at expiry.
- LOS 2 Contrast arbitrage and replication for forward commitments vs. contingent claims.** Forwards: symmetric payoff, one static replication (borrow $X(1+r)^{-T}$, buy the asset), exact price. Options: asymmetric payoff gives lower and upper no-arbitrage bounds; replication borrows (call) or lends (put) only a *proportion* of $PV(X)$ linked to moneyness and must be adjusted over time.
- LOS 3 Identify the factors that determine option value and their effects:** underlying price, exercise price, time to expiration, risk-free rate, volatility, income or cost of holding the underlying.

Key formulas

European options, no income on the underlying, discrete compounding (curriculum):

$$\text{Call exercise value} = \max(0, S_t - X(1+r)^{-(T-t)}), \quad \text{Put exercise value} = \max(0, X(1+r)^{-(T-t)} - S_t)$$

$$\max(0, S_t - X(1+r)^{-(T-t)}) < c_t \leq S_t, \quad \max(0, X(1+r)^{-(T-t)} - S_t) < p_t \leq X$$

Option price = exercise value + time value; time value = c_t – call exercise value (or p_t – put exercise value) > 0 , falling to zero at T .

Increase in	Call	Put	Why
Value of the underlying	+	–	Same direction as long/short forward
Exercise price	–	+	Call pays $S_T - X$, put pays $X - S_T$
Time to expiration	+	+ / –	More dispersion; but a deep ITM put waits longer for X
Risk-free rate	+	–	Lower $PV(X)$; does not directly affect time value
Volatility	+	+	More upside, downside still capped at premium
Income / cost of holding	– / +	+ / –	Income accrues to the asset owner, not the option holder

Worked example (CFA curriculum)

Put, one year, $X = \text{EUR } 1,000$, $r = 1\%$, six months left, $S_t = \text{EUR } 950$.

Exercise value = $1,000(1.01)^{-0.5} - 950 = \text{EUR } 45.04$. Moneyness: in the money by $1,000 - 950 = \text{EUR } 50$. If $p_t = \text{EUR } 50$, time value = $50 - 45.04 = \text{EUR } 4.96$.

Call bounds, $X = \text{EUR } 1,000$, $r = 1\%$, six months left, $S_t = \text{EUR } 1,050$.

Lower bound = $\max(0, 1,050 - 1,000(1.01)^{-0.5}) = \text{EUR } 54.96$; upper bound = $S_t = \text{EUR } 1,050$.

Call, GBP: $X = 50$, $r = 2\%$, three months left, $S_t = 57.50$, $c_t = 10$: exercise value = $57.50 - 50(1.02)^{-0.25} = 7.75$, time value = 2.25 (not $7.50/2.50$, which ignores PV of X). An OTM put ($S_t = 55$, $p_t = 5$) is **all time value**.

What examiners like to test

- Two definitions:** LM8 builds exercise value on $PV(X)$, while moneyness (and LM2's "intrinsic value $S_t - X$ ") use X itself.
[Note: the curriculum uses both; for exercise value/time value questions use $PV(X)$, for ITM/OTM use X]
- Same vs. opposite effects:** volatility and (usually) time raise both calls and puts; S , X , r and income/cost push calls and puts in opposite directions.
- Replication:** call = borrow and buy the underlying; put = short the underlying and lend; both need rebalancing, unlike a forward.

Module 9 — Option Replication Using Put–Call Parity

Topic: Derivatives · Exam relevance: High-yield · Calculation-heavy

The big picture

Two portfolios that pay exactly the same at expiry must cost the same today. A **protective put** (share + put) and a **fiduciary call** (call + risk-free bond paying X) both pay $\max(S_T, X)$, so their prices must match: that is put–call parity. Rearranging it lets you build any one of the four pieces from the other three, spot mispricing, and even see equity as a call on the firm's assets. Replacing the share with a forward plus a bond gives put–call *forward* parity.

Learning outcomes in plain language

- LOS 1 Explain put–call parity for European options.** Protective put = fiduciary call (same X and expiry, no income on the underlying); rearrange to price or replicate a put, call, share or bond; exploit a violation (buy the cheap side, sell the dear side, profit today, zero cash flow at T); covered call $\approx$ long bond + short put; firm-value application (equity = call on firm assets, risky debt = risk-free debt – put, the put $\approx$ credit spread).
- LOS 2 Explain put–call forward parity.** Replace S_0 by a synthetic asset, a long forward plus a bond paying $F_0(T)$, i.e. $F_0(T)(1+r)^{-T}$: synthetic protective put = fiduciary call; long put + short call = long bond + short forward.

Key formulas

European options, same X and T , discrete compounding as in the curriculum:

$$S_0 + p_0 = c_0 + X(1+r)^{-T} \quad \Rightarrow \quad p_0 = c_0 + X(1+r)^{-T} - S_0, \quad S_0 = c_0 - p_0 + X(1+r)^{-T}$$

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T} \quad \Rightarrow \quad p_0 - c_0 = [X - F_0(T)](1+r)^{-T}$$

Firm value: $V_0 = c_0 + PV(D) - p_0$; equity payoff = $\max(0, V_T - D)$, debt payoff = $\min(V_T, D) = D - \max(0, D - V_T)$

$X(1+r)^{-T}$ = price of a risk-free zero paying X at T ; D = face value of zero-coupon debt; V = firm asset value.

Replicate a long...	Underlying	Bond	Call	Put
Underlying S_0	—	Long	Long	Short
Risk-free bond	Long	—	Short	Long
Call c_0	Long	Short	—	Long
Put p_0	Short	Long	Long	—

Worked example (CFA curriculum)

Parity price (Biomian, VFO). $S_0 = \text{INR } 295$, $X = \text{INR } 265$, six months, $r = 4\%$, $c_0 = \text{INR } 59$. Bond = $265(1.04)^{-0.5} = 259.85$. $p_0 = 59 + 259.85 - 295 = \text{INR } 23.85$.

Arbitrage. If the put trades at INR 30: $295 + 30 > 59 + 259.85$. Sell the share and the put, buy the call and the bond: inflow today $295 + 30 - 59 - 259.85 = \text{INR } 6.15$; at T the positions net to zero in every scenario.

Covered call. $X = \text{INR } 325$, $p_0 = \text{INR } 56$: $S_0 - c_0 = 325(1.04)^{-0.5} - 56 \Rightarrow c_0 = \text{INR } 32.31$.

Forward parity. $F_0(T) = 295(1.04)^{0.5} = 300.84$; $p_0 = 59 + (265 - 300.84)(1.04)^{-0.5} = \text{INR } 23.85$ [Note: the book prints INR 23.86 here, a rounding difference; exact working gives 23.85, the same as spot parity].

What examiners like to test

- **Signs in the building blocks:** a long share = long call + short put + long bond; a short put = long share + short bond + short call. Write the parity equation first, then move terms.
- **Synthetic protective put** = long forward + long bond (face $F_0(T)$) + long put; it equals a fiduciary call.
- **Firm value:** shareholders $\approx$ long call on firm assets (strike D); debtholders $\approx$ risk-free debt + sold put. A more valuable put means more credit risk.

Module 10 — Valuing a Derivative Using a One-Period Binomial Model

Topic: Derivatives · Exam relevance: **High-yield** · **Calculation-heavy**

The big picture

Forwards can be priced without any view of where the underlying will go; options cannot, because their payoff is one-sided. The binomial model makes the simplest possible assumption — the price goes either up or down by known amounts — and then combines the option with h units of the underlying so the portfolio pays the same in both states. A riskless portfolio must earn the risk-free rate, which pins down the option price. The same answer comes from **risk-neutral** probabilities, and real-world probabilities never enter.

Learning outcomes in plain language

- LOS 1 Explain how to value a derivative with a one-period binomial model.** Build $S_1^u = R_u S_0$ and $S_1^d = R_d S_0$; compute option payoffs in each state; hedge ratio h^* ; hedged portfolio (sell 1 call, buy h^* shares) has the same value V_1 in both states; discount V_1 at r ; solve for c_0 (or p_0 , or use put–call parity). A wider $R_u - R_d$ (higher volatility) raises both call and put values.
- LOS 2 Describe risk neutrality.** Option value = expected payoff using risk-neutral probability π , discounted at the risk-free rate. π depends only on R_u , R_d and r — not on the actual probability q of an up move, the expected return, or investors' risk aversion.

Key formulas

$$R_u = \frac{S_1^u}{S_0} > 1, \quad R_d = \frac{S_1^d}{S_0} < 1, \quad h^* = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}, \quad V_0 = hS_0 - c_0 = \frac{V_1}{1+r} \Rightarrow c_0 = hS_0 - V_1(1+r)^{-1}$$

$$\pi = \frac{1+r-R_d}{R_u-R_d}, \quad c_0 = \frac{\pi c_1^u + (1-\pi)c_1^d}{(1+r)^T}, \quad p_0 = \frac{\pi p_1^u + (1-\pi)p_1^d}{(1+r)^T}$$

$c_1^u = \max(0, S_1^u - X)$, $c_1^d = \max(0, S_1^d - X)$ (puts: $\max(0, X - S_1)$); $V_1 = hS_1^u - c_1^u = hS_1^d - c_1^d$; r = risk-free rate (discrete compounding, as in the curriculum). For a put the hedge ratio is negative: buy the put *and* buy $|h^*|$ shares, $V_1 = |h^*|S_1 + p_1$.

Arbitrage: if the market call price is above the model price, borrow at r , sell the call, buy h^* shares $\Rightarrow$ riskless return above r .

Worked example (CFA curriculum)

Call, $S_0 = \text{EUR } 80$, $X = \text{EUR } 100$, **one year**, $r = 5\%$, $S_1^u = 110$, $S_1^d = 60$ ($R_u = 1.375$, $R_d = 0.75$). Payoffs $c_1^u = 10$, $c_1^d = 0$; $h^* = 10/50 = 0.20$. $V_1 = 0.20 \times 110 - 10 = 0.20 \times 60 = \text{EUR } 12$; $c_0 = 0.20 \times 80 - 12/1.05 = 16 - 11.43 = \text{EUR } 4.57$. Risk-neutral: $\pi = (1.05 - 0.75)/(1.375 - 0.75) = 0.48$; $c_0 = 0.48 \times 10/1.05 = \text{EUR } 4.57$ — same answer.

Highest Capital, $S_0 = \$50$, $X = \$55$, $r = 5\%$, **one year**. $\pm 20\%$: $S_1 = 60/40$, $h^* = 5/20 = 0.25$, $V_1 = \$10$, $V_0 = 10/1.05 = 9.52$, $c_0 = 12.50 - 9.52 = \$2.98$; $\pi = (1.05 - 0.8)/0.4 = 0.625$, $c_0 = 0.625 \times 5/1.05 = \$2.98$, $p_0 = 0.375 \times 15/1.05 = \$5.36$ (parity check: $50 + 5.36 = 2.98 + 55/1.05 = 55.36$). $\pm 40\%$: $h^* = 15/40 = 0.375$, $c_0 = 18.75 - 10.71 = \$8.04$, $p_0 = \$10.42$ — more volatility, higher option values. A more optimistic view of q changes **nothing**.

What examiners like to test

- Real vs. risk-neutral probabilities:** changing the actual chance of an up move does not change the price; changing R_u , R_d or r does. Up 25%/down 25% from \$16 with $r = 5\%$: $\pi = 0.60$.
- Put hedge:** h^* is negative (-0.75 for $X = £21$, $S_0 = £20$, $\pm 10\%$), so buy both the put and shares; $V_1 = £16.50$, $p_0 = 16.50(1.04)^{-0.5} - 15 = £1.18$.
- Negative rates still work:** $S_0 = \text{JPY } 8,000$, up 5%/down 15%, $r = -0.20\%$ for the period, put $X = 7,950$: $\pi = (0.998 - 0.85)/0.20 = 0.74$, $p_0 = 0.26 \times 1,150/0.998 = \text{JPY } 299.60$.
- Rate effect:** a higher r raises π and lowers the PV of payoffs, so it lowers put values (and raises calls).