

FRM® PART I · BOOK 1

Foundations of Risk Management

Detailed Formula Sheet

Every formula with variable definitions, intuition, worked examples and exam traps · Prepared by **RBei Classes**, Noida

What this book tests. Foundations carries about 20% of the Part I exam. Most questions are *conceptual* (risk types, governance, hedging, crisis lessons, GARP Code), but the **portfolio theory & CAPM** and **performance-measure** formulas are the calculation core — expect several numerical questions from them.

How to read this sheet. Blue boxes = formulas Green boxes = fully worked examples (every step verified) Orange boxes = exam traps Purple boxes = must-know numbers & facts.

Notation. R_F or r = risk-free rate; $E(R)$ = expected return; σ = standard deviation; σ^2 = variance; ρ = correlation; β = beta; w = portfolio weight; M = market portfolio; P = portfolio; B = benchmark. All returns are in decimal or % — just be consistent.

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1. The Building Blocks of Risk Management

Risk vs. risk taking, expected vs. unexpected loss, VaR, ES, economic capital, RAROC, risk types

1.1 Expected Loss (EL)

$$EL = PD \times LGD \times EAD \qquad LGD = 1 - RR$$

WHERE PD = probability of default; LGD = loss given default (% of exposure lost); EAD = exposure at default (amount owed); RR = recovery rate.

Intuition: EL is the *average*, predictable loss — a cost of doing business. Banks price it into the loan spread (like bad-debt expense). It is **not** what capital is held for.

1.2 Unexpected Loss (UL), Economic Capital & the loss distribution

$$UL = \text{Actual loss} - EL \qquad \text{Economic capital} \approx \text{Loss at chosen confidence level (VaR)} - EL$$

EL is covered by pricing & provisions; UL (the variability of loss above EL) is covered by **capital**. Correlation risk (e.g. defaults clustering in a recession, or default rate rising while collateral value falls) pushes losses into the UL region.

Worked example — EL on a loan

A bank lends INR 10 crore to a mid-size company. $PD = 2\%$, recovery rate = 40%.

$LGD = 1 - 0.40 = 0.60 \Rightarrow EL = 0.02 \times 0.60 \times 10 \text{ cr} = \text{INR } 0.12 \text{ crore (INR 12 lakh)}$.

Check: expected recovery if default = $40\% \times 10 = 4 \text{ cr}$; loss if default = 6 cr; $\times PD 2\% = 0.12 \text{ cr} \checkmark$

Worked example — Recovery value & LGD (counterparty)

A counterparty owes a net gain of 500,000 on a futures position but can pay only 400,000.

Recovery value = 400,000 $\Rightarrow RR = 400/500 = 80\%$; loss = 100,000 $\Rightarrow LGD = 20\%$.

1.3 Value at Risk (VaR) & Expected Shortfall (ES) — concepts used in this book

$VaR_{X\%,T}$: the loss that will *not* be exceeded with $X\%$ confidence over horizon T . Equivalently, there is a $(1 - X)\%$ chance of losing *more* than VaR.

$$\text{Historical VaR} = \text{the } (1 - X) \text{ percentile return} \times \text{Position value}$$

Expected Shortfall (ES / CVaR / expected tail loss): the *average* loss given that the loss exceeds VaR. Always $ES \geq VaR$.

VaR tells you the threshold, not the size of tail losses; it can be altered by changing the horizon or confidence level and has several versions.

ES captures tail magnitude.

Worked example — Historical VaR

1,000 monthly returns on a stock; the 5th-percentile return (the 50th worst) is -15.5% . Position = INR 1 crore.
 95% one-month VaR = $15.5\% \times 1 \text{ cr} = \text{INR } 15.5 \text{ lakh}$. *Meaning:* 5% chance of losing more than INR 15.5 lakh in any month.

1.4 Risk-Adjusted Return on Capital (RAROC)

$$\text{RAROC} = \frac{\text{Reward}}{\text{Risk}} = \frac{\text{After-tax net risk-adjusted expected return}}{\text{Economic capital}}$$

“Net risk-adjusted” = the expected return is already reduced by expected losses (EL). **Decision rule:** accept / expand an activity only if $\text{RAROC} > \text{cost of equity (hurdle rate)}$.

Four uses: (1) business-line comparison, (2) investment analysis (e.g. new credit product), (3) pricing strategy, (4) risk-management cost/benefit (e.g. is insurance worth it?).

Worked example — RAROC decision

A lending unit: revenue 30, operating cost 8, EL 4, tax 30%. Economic capital allocated = 100 (all INR crore). Cost of equity = 12%.
 Pre-tax risk-adjusted return = $30 - 8 - 4 = 18$; after tax = $18 \times (1 - 0.30) = 12.6$.
 $\text{RAROC} = 12.6/100 = 12.6\% > 12\% \Rightarrow$ the unit **adds value**. (If EC were 110, $\text{RAROC} = 11.45\% < 12\% \Rightarrow$ destroys value.)

EXAM TRAP: Capital is held for **unexpected** loss, not expected loss. EL is “priced in”. A question saying “capital covers expected losses” is wrong.

Must-know lists — Reading 1

Risk management process	(1) Identify → (2) Analyse & measure → (3) Evaluate impact → (4) Manage (avoid / retain / mitigate / transfer).
Loss spectrum	Expected (known) → Known unknowns (Knightian uncertainty) → Unknown unknowns (tail risk).
Quantitative tools	VaR, ES, economic capital. Qualitative: scenario analysis, stress testing (one factor), reverse stress testing.
Three lines of defence	1st: front-line / business units; 2nd: risk-management function; 3rd: independent internal audit.
Market risk	Interest-rate (incl. basis risk), equity (general vs. specific), FX, commodity.
Credit risk	Default, bankruptcy, downgrade, counterparty (incl. settlement / Herstatt) risk.
Liquidity risk	Funding liquidity (cannot meet obligations/refinance) vs. market/trading liquidity (cannot sell without big discount).
Non-financial	Operational, legal & regulatory, business & strategic, reputation.

2. How Do Firms Manage Financial Risk?

Strategies, risk appetite, risk mapping, hedging pros & cons, instruments

2.1 Risk appetite vs. capacity (the only “formula” here)

$$\text{Accepted risk} < \text{Risk appetite} < \text{Risk capacity}$$

Risk appetite = the level/types of risk a firm is *willing* to retain (willingness) within what it is *able* to bear (ability, capped by internal controls & regulation). Leave a margin for estimation error.

Worked example — Setting risk limits

Total risk capacity = 200; board sets risk appetite = 170; managers should cap actual exposure somewhat below, e.g. **150**, to leave room for model/estimation error.

Must-know — Reading 2

4 strategies	Accept (retain) — small impact / owners want exposure / can pass cost on; Avoid — exit the activity; Mitigate — reduce frequency/severity (collateral, automation); Transfer — insurance/derivatives (adds counterparty risk).
5-step process	Identify risk appetite → Map risks → Operationalise appetite → Implement → Monitor & adjust.
Risk mapping needs	Magnitude, timing, location, calculation method (current vs. projected), netting potential.
Hedging — for	Lower cost of capital, higher debt capacity, fewer covenants, tax smoothing, signalling, easier planning, lock in margins, cheaper than insurance.
Hedging — against (theory)	Modigliani–Miller (perfect markets ⇒ hedging adds no value); CAPM (only systematic risk matters); zero-sum game. <i>All ignore costs of financial distress.</i>
Hedging — against (practice)	Loss of focus, compliance/disclosure costs, complexity & leverage, new unintended risks, reveals private info.
Challenges	Mis-mapped exposures, dynamic markets, poor communication (Metallgesellschaft), lack of skills.
Static vs. dynamic	Static: set once, match position. Dynamic: rebalanced as exposure changes — more cost & monitoring.
Exchange vs. OTC	Exchange: liquid, cheap, low counterparty risk, but standardised ⇒ basis risk . OTC: customised, but illiquid, costlier, counterparty risk.

EXAM TRAP: Buying forwards/futures/options/insurance = **transfer**; changing the business itself (collateral, automation) = **mitigate**. Debt-holders prefer *less* risk; shareholders may accept more.

3. The Governance of Risk Management

Post-crisis regulation, board duties, committees, risk appetite vs. strategy, audit

Numbers & facts to memorise — Reading 3

Basel I (1988)	Minimum capital = 8% of risk-weighted assets (credit risk only).
Basel II (2006)	Adds trading book/market risk, disclosure (market discipline) and supervisory review.
Basel III	Tier 1 limited to common equity & retained earnings. $\text{LCR} = \frac{\text{HQLA}}{\text{net cash outflows over 30 days}} \geq 100\%; \quad \text{NSFR} = \frac{\text{Available stable funding}}{\text{Required stable funding}} \geq 100\% \text{ (1-year)}.$
Macroprudential overlay	Leverage ratio $\frac{\text{Tier 1 capital}}{\text{Total exposure}} \geq 3\%$; countercyclical buffer; TLAC for G-SIBs; central clearing; better tail-risk modelling.
FRTB (2016)	Fundamental Review of the Trading Book — market-risk capital.
SOX (2002)	CEO & CFO certify financial statements; independent, financially literate audit committee.
Dodd–Frank (2010)	Fed oversight of SIFIs (>USD 50 bn); orderly liquidation; living wills ; Volcker rule (no proprietary trading); CFPB; stress tests: DFAST (>USD 10 bn), CCAR (>USD 50 bn).
G20 pay reforms	No multi-year guaranteed bonuses; deferral & clawbacks; variable pay capped at 100% of salary (200% with shareholder approval); disclosure; independent compensation committee.
Bonus bond (UBS)	Forfeited if regulatory capital ratio falls below 7.5% .
Board best practice	Majority independent; CEO ≠ Chair (MF Global lesson); CRO reports to CEO with direct access to board; risk committee; independent audit committee; risk advisory director.
Who does what	Board/risk committee set appetite; CRO monitors limits & approves temporary excesses; business units implement; finance & operations execute mitigation/transfer; internal audit reports to audit committee.

4. Credit Risk Transfer Mechanisms

CDS, CDO, CLO, traditional mitigants, securitisation, OTD model

Must-know — Reading 4

CDS	Protection buyer pays periodic (quarterly) premium; seller pays if reference entity defaults. Pure price of credit risk.
CDO / CMO / CDO²	Pooled loans tranching: equity (first loss, highest yield, “toxic waste”) → mezzanine → super-senior (paid first, AAA, lowest yield). CMO = only mortgages; CDO ² = CDO of CDO tranches.
CLO	Like a CDO but backed mainly by underwritten bank loans — held up better after the crisis.
Traditional mitigants	Third-party insurance/guarantees, exposure netting , marking-to-market , collateral (beware wrong-way risk), termination clauses, reassignment, syndication (lead bank keeps ≈20%).
Wrong-way risk	Collateral value falls exactly when default risk rises (e.g. oil firm pledging oil).
Securitisation steps	(1) Create SPV (off-balance-sheet) → (2) SPV buys loans → (3) tranche by seniority/rating → (4) sell to investors.
Buy-and-hold vs. OTD	Buy-and-hold: bank keeps loan, earns interest ⇒ incentive for quality. Originate-to-distribute : earns fees ⇒ volume focus, moral hazard, opacity.
Crisis numbers	CDS notional ≈ USD 45 trillion (2007); ≈USD 400 bn of Lehman’s ≈USD 600 bn debt had CDS protection.
Reforms	Dodd–Frank/Volcker; CFTC regulates swaps; SEC Section 15G : originators must retain ≥5% of credit risk (cannot hedge it).

EXAM TRAP: Credit derivatives did not *cause* the crisis; their **misuse** (speculation, poor underwriting in OTD) did. A call feature is **not** a credit-risk-transfer tool.

5. Modern Portfolio Theory & the CAPM

Diversification, efficient frontier, CML, beta, SML/CAPM, and performance measures — the calculation core of this book

5.1 Expected return & risk of a two-asset portfolio

$$E(R_P) = w_A E(R_A) + w_B E(R_B), \quad w_A + w_B = 1$$

$$\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \rho_{AB} \sigma_A \sigma_B \quad \text{Cov}(A, B) = \rho_{AB} \sigma_A \sigma_B$$

n assets: $\sigma_P^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \rho_{ij} \sigma_i \sigma_j$ Return is a weighted average; risk is **not** (unless $\rho = +1$). The lower ρ , the greater the diversification benefit. With $\rho = +1$: $\sigma_P = w_A \sigma_A + w_B \sigma_B$.

5.2 Minimum-variance & zero-risk portfolio

$$w_A^* = \frac{\sigma_B^2 - \rho_{AB} \sigma_A \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2\rho_{AB} \sigma_A \sigma_B} \quad (\text{global minimum-variance weight})$$

$$\text{If } \rho_{AB} = -1: \quad w_A = \frac{\sigma_B}{\sigma_A + \sigma_B} \Rightarrow \sigma_P = 0 \text{ (a synthetic risk-free asset)}$$

The global minimum-variance portfolio is the *leftmost* point of the efficient frontier. Portfolios below it are inefficient; portfolios above the frontier are unattainable.

Worked example — Portfolio risk & diversification

$w_A = 60\%$, $w_B = 40\%$, $\sigma_A = 20\%$, $\sigma_B = 30\%$, $\rho_{AB} = 0.3$.

$$\sigma_P^2 = (0.6)^2(0.20)^2 + (0.4)^2(0.30)^2 + 2(0.6)(0.4)(0.3)(0.20)(0.30) \\ = 0.0144 + 0.0144 + 0.00864 = 0.03744 \Rightarrow \sigma_P = \sqrt{0.03744} = \mathbf{19.35\%}.$$

Weighted-average risk would be $0.6(20) + 0.4(30) = 24\% \Rightarrow$ diversification saves ≈ 4.65 pp.

If $\rho = -1$: $w_A = \frac{0.30}{0.20+0.30} = 60\%$, and $\sigma_P = 0.6(0.20) - 0.4(0.30) = 0$ ✓ (check with the general formula: $\frac{0.09+0.06}{0.04+0.09+0.12} = \frac{0.15}{0.25} = 0.60$ ✓)

5.3 Capital Market Line (CML) — efficient portfolios only

$$E(R_P) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M} \right] \sigma_P$$

WHERE intercept = R_F ; slope = $\frac{E(R_M) - R_F}{\sigma_M}$ = market price of risk = **Sharpe ratio of the market**; risk measure = **total risk** σ_P .

Every investor holds a mix of the risk-free asset and the market (tangency) portfolio M . Left of M = lending (part in T-bills); right of M = borrowing at R_F (leverage). Portfolio on the CML with weight w in M : $E(R_P) = R_F + w[E(R_M) - R_F]$, $\sigma_P = w\sigma_M$.

Worked example — Using the CML

$R_F = 7\%$, $E(R_M) = 13\%$, $\sigma_M = 20\%$. Investor wants $\sigma_P = 10\%$.

Slope = $(13 - 7)/20 = 0.30 \Rightarrow E(R_P) = 7 + 0.30 \times 10 = 10\%$. This needs $w = \sigma_P/\sigma_M = 50\%$ in M and 50% in T-bills.

Check: $7 + 0.5(13 - 7) = 10\% \checkmark$. For $\sigma_P = 30\%$: $w = 150\%$ (borrow 50% at R_F), $E(R_P) = 7 + 1.5 \times 6 = 16\%$.

5.4 Beta — systematic risk

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2} = \frac{\rho_{i,M} \sigma_i \sigma_M}{\sigma_M^2} = \rho_{i,M} \frac{\sigma_i}{\sigma_M}$$

$$\beta_M = \frac{\text{Cov}(R_M, R_M)}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1, \quad \beta_{R_F} = 0, \quad \beta_P = \sum_i w_i \beta_i$$

Regression estimate: $(R_{it} - R_{Ft}) = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + e_{it}$ — beta is the slope of the characteristic line. $\beta > 1$: aggressive/cyclical (e.g. luxury goods); $\beta < 1$: defensive (e.g. utilities). Only systematic risk is rewarded because specific risk can be diversified away for free.

Worked example — Two ways to compute beta

$\sigma_M = 20\%$, $\sigma_A = 30\%$, $\rho_{A,M} = 0.8$. (a) $\beta_A = 0.8 \times \frac{0.30}{0.20} = 1.20$.

(b) Given $\text{Cov}(A, M) = 0.048$: $\beta_A = \frac{0.048}{0.20^2} = \frac{0.048}{0.04} = 1.20$. Consistent: $0.8 \times 0.30 \times 0.20 = 0.048 \checkmark$

5.5 CAPM & the Security Market Line (SML)

$$E(R_i) = R_F + \beta_i [E(R_M) - R_F]$$

WHERE $E(R_M) - R_F$ = market risk premium (MRP) = slope of the SML; intercept = R_F (at $\beta = 0$); the market plots at ($\beta = 1$, $E(R_M)$).

Derivation in one line: $E(R_i)$ is linear in β : $E(R_i) = a + m\beta_i$. At $\beta = 0$, $a = R_F$; at $\beta = 1$, $a + m = E(R_M) \Rightarrow m = E(R_M) - R_F$.

Valuation rule: CAPM gives the *required* (hurdle) return.

- Forecast return $>$ CAPM required return $\Rightarrow$ plots **above** SML $\Rightarrow$ **undervalued** (buy), positive alpha.
- Forecast return $<$ CAPM required return $\Rightarrow$ plots **below** SML $\Rightarrow$ **overvalued** (sell/short).

Worked example — CAPM required return & mispricing

$R_F = 7\%$, $\text{MRP} = E(R_M) - R_F = 6\%$, $\beta = 1.25$ (e.g. a cyclical NSE stock).

Required return = $7 + 1.25 \times 6 = 14.5\%$. Analyst forecasts 16% $\Rightarrow$ alpha = $16 - 14.5 = +1.5\% \Rightarrow$ above SML, **undervalued**.

Reverse problem: a fairly-valued stock has $\beta = 1.5$, $E(R) = 13\%$, $R_F = 4\%$. Then $13 = 4 + 1.5 \text{MRP} \Rightarrow \text{MRP} = 6\%$, so $E(R_M) = 4 + 6 = 10\%$.

EXAM TRAP: The **CML** uses σ (total risk) and applies only to *efficient portfolios*. The **SML** uses β (systematic risk) and prices *any* asset or portfolio. An individual stock does not lie on the CML.

CAPM assumptions (know them all)

Information freely available; frictionless markets (no taxes, commissions, transaction costs); assets infinitely divisible; perfect competition (investors are price-takers); decisions based only on mean & variance (skewness/kurtosis ignored); unlimited borrowing & lending at R_F ; homogeneous expectations over a single period.

5.6 Risk-adjusted performance measures

Measure	Formula	Risk used / interpretation
Sharpe (SPI)	$\frac{E(R_P) - R_F}{\sigma_P}$	Total risk. Use for a portfolio that is the investor's <i>entire</i> holding / not well diversified. Market's Sharpe = CML slope.
Treynor (TPI)	$\frac{E(R_P) - R_F}{\beta_P}$	Systematic risk. Use for well-diversified portfolios. Market's Treynor = $E(R_M) - R_F$ = SML slope.
Jensen's alpha (JPI)	$\alpha_P = E(R_P) - \{R_F + \beta_P[E(R_M) - R_F]\}$	Excess over CAPM. $\alpha > 0$ superior; = 0 in equilibrium. Best for comparing portfolios with the <i>same</i> beta.
Tracking error	$TE = \sigma(R_P - R_B)$	Std. dev. of <i>active return</i> ; "active risk". Index fund $TE \approx 0$.
Information ratio	$IR = \frac{E(R_P) - E(R_B)}{TE}$	Active return per unit of active risk.
Sortino	$\frac{E(R_P) - R_{MIN}}{DD}$	Uses only downside risk below a minimum acceptable return R_{MIN} (MAR/target T).

Downside deviation (GARP): $DD = \sqrt{\frac{1}{N} \sum_{t=1}^N [\min(0, R_{Pt} - T)]^2}$

Treynor–Jensen link: dividing the regression by β_P : $\frac{\alpha_P}{\beta_P} = TPI_P - [E(R_M) - R_F] \Rightarrow$ if $\beta_P > 0$, superior Treynor $\Leftrightarrow$ positive alpha (rankings may still differ).

Worked example — Sharpe, Treynor, Jensen vs. the market

$E(R_P) = 14\%$, $\sigma_P = 25\%$, $\beta_P = 1.1$; $E(R_M) = 12.5\%$, $\sigma_M = 20.2\%$; $R_F = 2.6\%$.

	Portfolio	Market
Sharpe	$(14 - 2.6)/25 = \mathbf{0.456}$	$(12.5 - 2.6)/20.2 = \mathbf{0.490}$
Treynor	$(14 - 2.6)/1.1 = \mathbf{10.36\%}$	$(12.5 - 2.6)/1 = \mathbf{9.90\%}$
Jensen α	$14 - [2.6 + 1.1(9.9)] = 14 - 13.49 = \mathbf{+0.51\%}$	0

Interpretation: beats the market on Treynor & alpha (systematic risk) but loses on Sharpe (total risk) $\Rightarrow$ the portfolio carries unrewarded **specific risk** — it is **not well diversified**.

Link check: $\alpha/\beta = 0.51/1.1 = 0.464\%$ and $TPI - MRP = 10.364 - 9.9 = 0.464\%$ ✓

Worked example — Information ratio & Sortino

Active portfolio 15%, benchmark (e.g. Nifty 50 TRI) 12%, tracking error 9%, $R_{MIN} = 4\%$, downside deviation 7%.

$IR = (15 - 12)/9 = \mathbf{0.333}$ Sortino = $(15 - 4)/7 = \mathbf{1.571}$

Worked example — Computing downside deviation

Quarterly returns: +8%, -2%, +5%, -6%; target $T = 0\%$. Shortfalls $\min(0, R - T)$: 0, -2, 0, -6.

$DD = \sqrt{(0 + 4 + 0 + 36)/4} = \sqrt{10} = \mathbf{3.162\%}$. Mean = $(8 - 2 + 5 - 6)/4 = 1.25\% \Rightarrow$ Sortino = $1.25/3.162 = \mathbf{0.395}$.

EXAM TRAP: (1) Sharpe/Treynor/Jensen/IR: *higher is better*. (2) Treynor > market but Sharpe < market $\Rightarrow$ **poorly diversified**. (3) The IR numerator is excess over the **benchmark**, not over R_F . (4) Sortino replaces R_F with R_{MIN} and σ with downside deviation. (5) If $R_{MIN} = R_F$, Sortino $\geq$ Sharpe whenever $DD < \sigma$.

6. Arbitrage Pricing Theory & Multifactor Models

APT, factor surprises, single- vs. multi-factor, factor hedging, Fama–French

6.1 Multifactor (APT) return-generating model

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \cdots + \beta_{iK}F_K + e_i, \quad F_k = I_k - E(I_k)$$

WHERE R_i = realised return; $E(R_i)$ = expected return; F_k = **surprise** in factor k (actual – expected); β_{ik} = factor beta/loading/sensitivity; e_i = firm-specific (idiosyncratic) return, $E(e_i) = 0$. Only *unexpected* changes move returns away from $E(R_i)$ — expected changes are already priced in.

6.2 APT expected return (pricing equation)

$$E(R_P) = R_F + \beta_{P1}RP_1 + \beta_{P2}RP_2 + \cdots + \beta_{PK}RP_K, \quad RP_k = E(I_k) - E(R_Z)$$

RP_k = risk premium on factor k ; $E(R_Z)$ = return on the zero-beta portfolio ($= R_F$). Holds for well-diversified portfolios (and nearly all securities), otherwise arbitrage exists.

APT assumptions (only three): returns driven by systematic factors; specific risk diversifiable; **no arbitrage** among well-diversified portfolios. APT does *not* assume normality, mean–variance investors, or a market portfolio; it does *not* name the factors. CAPM is the special one-factor case. Chen–Roll–Ross factors: yield-curve spread, expected & unexpected inflation, industrial production, default (quality) spread.

Worked example — Single-factor then two-factor model

$E(R) = 10\%$. GDP beta = 2.0; expected GDP growth 3.2%, actual 2.6% $\Rightarrow$ surprise = -0.6% .

Single factor: $R = 10 + 2.0(-0.6) = \mathbf{8.8\%}$.

Add consumer-sentiment beta = 1.5; expected 1.0%, actual 0.75% $\Rightarrow$ surprise = -0.25% .

Two factors: $R = 10 - 1.2 + 1.5(-0.25) = 10 - 1.2 - 0.375 = \mathbf{8.425\%} \approx \mathbf{8.43\%}$ (closer to an actual 8.25%; the rest is e_i or a missing factor).

Worked example — APT expected return

$R_F = 4\%$; factor 1 premium 3%, $\beta_1 = 1.2$; factor 2 premium 2%, $\beta_2 = 0.5$. $E(R) = 4 + 1.2(3) + 0.5(2) = 4 + 3.6 + 1.0 = \mathbf{8.6\%}$.

6.3 Hedging factor exposures

Factor portfolio: a well-diversified portfolio with $\beta = 1$ to one factor and $\beta = 0$ to all others.

To neutralise a portfolio's exposures, short a combination of factor portfolios in the amounts of its betas, with the remainder in the risk-free asset:

$$\text{Hedge portfolio } H = \beta_{P1} \times FP_1 + \beta_{P2} \times FP_2 + \left(1 - \sum \beta_{Pk}\right) \times R_F$$

Long P , Short $H \Rightarrow$ zero factor exposure, zero net investment; arbitrage profit = $E(R_P) - E(R_H)$

Challenges: model risk (wrong factors/betas), rebalancing too often (costs) or too rarely (tracking error), non-stationarity in crises.

Worked example — Factor hedge / arbitrage

Portfolio 1: GDP beta 0.50, consumer-sentiment beta 0.30, $E(R_1) = 12\%$.

Hedge $H = 50\%$ GDP factor portfolio + 30% sentiment factor portfolio + 20% risk-free; $E(R_H) = 10\%$.

Long Portfolio 1, short H (equal amounts): factor betas $0.50 - 0.50 = 0$ and $0.30 - 0.30 = 0 \Rightarrow$ riskless profit = $12 - 10 = \mathbf{2\%}$. If instead $E(R_H) = 14\%$: long H , short Portfolio 1 $\Rightarrow 2\%$.

6.4 Fama–French three-factor model

$$E(R_i) = R_F + \beta_{i,M}[E(R_M) - R_F] + \beta_{i,SMB}E(SMB) + \beta_{i,HML}E(HML)$$

WHERE **SMB** (small minus big) = return on small-cap – large-cap stocks (size premium); **HML** (high minus low) = return on high book-to-market (value) – low book-to-market (growth) stocks. Extensions: Carhart (1997) adds **momentum**; Fama–French five-factor (2015) adds **RMW** (robust minus weak profitability) and **CMA** (conservative minus aggressive investment).

Worked example — Fama–French

$\beta_M = 0.85$, $\beta_{SMB} = 1.65$, $\beta_{HML} = -0.25$; equity risk premium 8.5%, SMB 2.5%, HML 1.75%, $R_F = 2.75\%$.

$E(R) = 2.75 + 0.85(8.5) + 1.65(2.5) - 0.25(1.75) = 2.75 + 7.225 + 4.125 - 0.4375 = \mathbf{13.66\%}$.

Reading it: $\beta_{SMB} > 0$ = small-cap tilt; $\beta_{HML} < 0$ = growth tilt. Any actual return above 13.66% is alpha.

EXAM TRAP: A negative factor beta is perfectly valid (it *reduces* expected return when that premium is positive). Adding factors captures more *systematic* risk but never removes firm-specific risk — only diversification does that.

7–10. Qualitative Readings: Data, ERM, Disasters, Crisis, Code

No calculations — but frequent concept questions. Key facts only.

Reading 7 — Risk Data Aggregation & Reporting (BCBS 239)

Benefits	Anticipate problems, find routes back to health in stress, better resolvability (esp. G-SIBs), better strategy & profitability.
Governance & infra	P1 Governance; P2 Data architecture & IT infrastructure.
Aggregation (P3–6)	Accuracy & integrity; Completeness; Timeliness; Adaptability — must be met <i>together</i> , not traded off.
Reporting (P7–11)	Accuracy; Comprehensiveness; Clarity & usefulness; Frequency (higher in stress); Distribution (with confidentiality).
Data models	Conceptual (most abstract), logical, physical (tables, keys), semantic.
Model risks	Input, estimation, valuation, hedging risk. Aim for a <i>single authoritative source</i> per risk type; largely automated aggregation.

Reading 8 — Enterprise Risk Management & scenario analysis

ERM vs. silo	ERM is integrated, top-down, enterprise-wide, linked to strategy; silos manage each risk separately and miss interactions.
Motivations	Better risk-adjusted decisions, capital allocation, lower earnings volatility, regulatory & rating-agency expectations, avoid surprises.
Strong risk culture	Tone from the top, accountability, effective challenge, incentives aligned with risk appetite, open communication of bad news.
Scenario analysis	Forward-looking, captures severity & correlations and “what-ifs”; but subjective, no probabilities, prone to bias. Used in stress testing & capital planning (e.g. CCAR).

Reading 9 — Financial disasters: the lesson in one line

US Savings & Loan (1980s)	Interest-rate risk: short-term deposits funding long-term fixed-rate mortgages.
Lehman, Continental Illinois, Northern Rock	Funding liquidity risk: reliance on short-term wholesale funding.
Metallgesellschaft (1993)	Hedging strategy & communication: stack-and-roll short-dated futures vs. long-dated supply contracts ⇒ huge margin calls, hedge unwound at the worst time.
Niederhoffer, LTCM, London Whale	Model risk: short volatility/tail exposure, leverage, liquidity & correlation breakdown, flawed VaR.
Barings (Nick Leeson)	Rogue trading: no separation of front and back office.
Bankers Trust, Orange County, Sachsen LB	Financial engineering: complex/opaque products, leverage, mis-selling.
Volkswagen	Reputation risk: emissions cheating.
Enron	Corporate governance: off-balance-sheet SPEs, accounting fraud.
SWIFT / Bangladesh Bank	Cyber risk.

Reading 10 — Great Financial Crisis (2007–09)

Build-up: low rates, housing boom, subprime & adjustable-rate mortgages, OTD securitisation, CDOs & CDO², over-optimistic ratings, leverage and reliance on short-term **repo & ABCP** funding (SIVs). Trigger: rate resets + falling house prices ⇒ defaults ⇒ run on whole-sale funding, haircuts rose, Lehman failure (Sept 2008). Central-bank response: lender of last resort, new lending facilities, lower rates, quantitative easing, capital injections & guarantees.

Reading 11 — GARP Code of Conduct

Principles: professional integrity & ethical conduct; conflicts of interest (disclose/avoid); confidentiality; **Professional Standards:** fundamental responsibilities and adherence to generally accepted risk practices.
Violations may lead to suspension or revocation of GARP membership and of the FRM designation (and loss of the right to use it).

11. One-Page Formula Recap

Tear-out summary for last-minute revision

Expected loss	$EL = PD \times LGD \times EAD, \quad LGD = 1 - RR$
Economic capital	$EC \approx \text{VaR}_{X\%} - EL$
RAROC	$\frac{\text{After-tax net risk-adjusted expected return}}{\text{Economic capital}} > \text{cost of equity}$
Portfolio return	$E(R_P) = \sum w_i E(R_i)$
Portfolio variance	$\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \rho_{AB} \sigma_A \sigma_B$
Min-variance weight	$w_A^* = \frac{\sigma_B^2 - \rho \sigma_A \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2\rho \sigma_A \sigma_B}$
CML	$E(R_P) = R_F + \frac{E(R_M) - R_F}{\sigma_M} \sigma_P$
Beta	$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2} = \rho_{iM} \frac{\sigma_i}{\sigma_M}, \quad \beta_P = \sum w_i \beta_i$
CAPM / SML	$E(R_i) = R_F + \beta_i [E(R_M) - R_F]$
Sharpe	$\frac{E(R_P) - R_F}{\sigma_P}$
Treynor	$\frac{E(R_P) - R_F}{\beta_P}$
Jensen's alpha	$E(R_P) - \{R_F + \beta_P [E(R_M) - R_F]\}$
Tracking error	$\sigma(R_P - R_B)$
Information ratio	$\frac{E(R_P) - E(R_B)}{TE}$
Sortino	$\frac{E(R_P) - R_{MIN}}{\sqrt{\frac{1}{N} \sum \min(0, R_{Pt} - R_{MIN})^2}}$
Multifactor model	$R_i = E(R_i) + \sum_k \beta_{ik} F_k + e_i \quad (F_k = \text{surprise})$
APT	$E(R_P) = R_F + \sum_k \beta_{Pk} RP_k$
Fama–French	$E(R_i) = R_F + \beta_M [E(R_M) - R_F] + \beta_{SMB} E(SMB) + \beta_{HML} E(HML)$
Key regulatory ratios	Capital $\geq 8\%$ RWA; Leverage $\geq 3\%$; LCR $\geq 100\%$ (30 days); NSFR $\geq 100\%$ (1 yr); retention $\geq 5\%$